**Day 38** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (2e^{2x} + \frac{3}{x})\,dx$.

$\int (2e^{2x} + \frac{3}{x})\,dx = e^{2x} + 3\ln x$

$= [e^{2x} + 3\ln x]_1^2$

$= \ln 8 + e^4 - e^2$

**Answer: $\ln 8 + e^4 - e^2$**

## 2 Union, conditional probability and independence (All boards)

P(A) = 0.4, P(B) = 0.2, P(A ∩ B) = 0.15.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) P(A ∪ B) = 0.4 + 0.2 − 0.15 = 0.45

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.2} = 0.75$

(c) P(A)P(B) = 0.4 × 0.2 = 0.08, which does not equal P(A ∩ B)

**Answer: (a) 0.45 (b) 0.75 (c) not independent**

## 3 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that 15 cos θ + 8 sin θ ≡ 17 cos(θ − 28.1°),
find the greatest value of $\frac{1}{(19 + 15\cos\theta + 8\sin\theta)}$.

Denominator is smallest when 17 cos(θ − 28.1°) = −17
Smallest denominator = 19 − 17 = 2

Greatest value = $\frac{1}{2}$

**Answer: $\frac{1}{2}$**

## 4 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 20, frequency 15, is a bar 4 cm wide, 1.875 cm tall.
(a) Find the bar height for width 20, frequency 24.
(b) Find the frequency of a 1 cm by 5 cm bar.

Area of bar ∝ frequency: 4 × 1.875 = 7.5 cm² represents 15
so 1 unit of frequency = 0.5 cm²
(a) Width = 4 cm, area = 0.5 × 24 = 12 cm², height = 12 ÷ 4 = 3 cm
(b) Area = 1 × 5 = 5 cm², frequency = 5 ÷ 0.5 = 10

**Answer: (a) 3 cm (b) 10**

## 5 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $9^{2x-3} = 3^{x+3}$.

Write both sides as powers of 3: $3^{2(2x-3)} = 3^{x+3}$
Equate the indices: 4x − 6 = x + 3
3x = 9
x = 3

**Answer: x = 3**

## 6 Velocity from variable acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A model car moves in a straight line. At time t seconds its acceleration is $5t + 4$ m s$^{-2}$.
When $t = 0$ the velocity is 4 m s$^{-1}$. Find the velocity when $t = 4$.

$v = \int (5t + 4)\,dt = \frac{5}{2}t^2 + 4t + c$

$t = 0$, $v = 4$ so $c = 4$

$t = 4$: $v = 60$

**Answer: 60 m s$^{-1}$**

## 7 Area under a curve (All boards)

The region R is bounded by the axes, the line $x = \ln 3$
and the curve $y = 2e^{2x} + 3$. Find the exact area of R.

$\text{Area} = \int_0^{\ln 3} (2e^{2x} + 3)\,dx = [3x + e^{2x}]_0^{\ln 3}$

$= \ln 27 + 8$

**Answer: ln 27 + 8**

## 8 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + px^2 + qx - 8$.
When f(x) is divided by $(x + 3)$ the remainder is 31, and when divided by $(x - 3)$ the remainder is 43. Find p and q.

$f(-3) = 31$: $-27 + 9p - 3q - 8 = 31$, so $9p - 3q = 66$

$f(3) = 43$: $27 + 9p + 3q - 8 = 43$, so $9p + 3q = 24$

Solving simultaneously: $p = 5$, $q = -7$

**Answer: p = 5, q = −7**

## 9 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 2t - 0.5t^2$ m s$^{-1}$, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 2 - t$

Maximum speed when $a = 0$: $t = 2 \div 1 = 2$

$v = 2 \times 2 - 0.5 \times 2^2$

**Answer: 2 m s$^{-1}$ at t = 2 s**

## 10 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 3x)^{-2}$ in ascending powers of x up to and including the term in $x^3$.
(b) State the range of values of x for which the expansion is valid.

$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Here $n = -2$ and $a = 3$:

coefficients: x: $-6$, $x^2$: 27, $x^3$: $-108$

Valid when $|3x| < 1$, that is $|x| < \frac{1}{3}$

**Answer: (a) $1 - 6x + 27x^2 - 108x^3 + \ldots$ (b) $|x| < \frac{1}{3}$**