Day 37 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when
$2x^3 - 11x^2 + 4x + 25$ is divided by $(x - 4)$.

$2x^2 \times (x - 4) = 2x^3 - 8x^2$, leaving $-3x^2 + 4x + 25$
$-3x \times (x - 4) = -3x^2 + 12x$, leaving $-8x + 25$
$-8 \times (x - 4) = -8x + 32$, leaving -7
Quotient $2x^2 - 3x - 8$, remainder -7
Answer: quotient $2x^2 - 3x - 8$, remainder -7

2 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 3 \cot 2x$ with respect to x,
and find the exact gradient when $x = \frac{\pi}{12}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = -6 \operatorname{cosec}^2 2x$
At $x = \frac{\pi}{12}$: $\frac{dy}{dx} = -24$
Answer: $\frac{dy}{dx} = -6 \operatorname{cosec}^2 2x$; gradient -24

3 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{-1 + 3\sqrt{7}}{3 + \sqrt{7}}$ in the form $p + q\sqrt{7}$, where p and q are integers.

Multiply top and bottom by the conjugate $(3 - \sqrt{7})$:
Denominator: $(3 + \sqrt{7})(3 - \sqrt{7}) = 3^2 - 1^2 \times 7 = 2$
Numerator: $(-1 + 3\sqrt{7})(3 - \sqrt{7}) = -24 + 10\sqrt{7}$
So the fraction $= \frac{-24 + 10\sqrt{7}}{2} = -12 + 5\sqrt{7}$
Answer: $-12 + 5\sqrt{7}$

4 Poisson distribution (CIE)

X ~ Po(1.2), modelling the number of calls to a helpline per minute.
Find, to 3 significant figures,
(a) $P(X \leq 1)$
(b) $P(X \geq 1)$

(a) $P(X \leq 1) = P(X = 0) + ... + P(X = 1) = 0.663$
(b) $P(X \geq 1) = 1 - P(X \leq 0) = 0.699$
Answer: (a) 0.663 (b) 0.699

5 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = x^2 e^{3 - 3x} + 2$.
Find the equation of the tangent to C at the point where x = 1, in the form $y = mx + c$.

$\frac{dy}{dx} = -x(3x - 2)e^{3 - 3x}$
At $x = 1$: $y = 3$ and $\frac{dy}{dx} = -1$
$y - 3 = -1(x - 1)$, so $y = -x + 4$
Answer: $y = -x + 4$

6 Inverse normal (All boards)

A battery lasts X hours, normal with mean 30, s.d. 2.
Find k (3 s.f.): (a) P(X < k) = 0.25,
(b) P(X > k) = 0.2.

(a) $z = -0.6745$, so $k = 30 + (-0.6745) \times 2 = 28.7$
(b) P(X < k) = 0.8, $z = 0.8416$, so $k = 30 + 0.8416 \times 2 = 31.7$
Answer: (a) 28.7 (b) 31.7

7 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected from ground level with initial velocity components 17 m s^{-1} horizontally and 9.8 m s^{-1} vertically upwards.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 9.8^2 - 2 \times 9.8 \times H$, $H = 96.04 \div 19.6$
(b) At the top the vertical velocity is zero, so the speed is the horizontal component.
Answer: (a) 4.9 m (b) 17 m s^{-1}

8 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations
to find the value of $\lim_{\theta \to 0} \frac{\tan 3\theta \sin 3\theta}{1 - \cos 3\theta}$.

$\tan 3\theta \sin 3\theta \approx 3\theta \times 3\theta = 9\theta^2$

$1 - \cos 3\theta \approx \frac{9\theta^2}{2}$

Ratio = $\frac{9\theta^2}{\frac{9\theta^2}{2}} = 2$

Answer: 2

9 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is (6, 4) and B is (0, −2).
Find an equation of the circle in the form
$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = (3, 1)
$\text{Radius}^2 = (6 - 3)^2 + (4 - 1)^2 = 18$
Answer: $(x - 3)^2 + (y - 1)^2 = 18$

10 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (4 kg) and B (12 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 1.25 m above the ground, then released.
Take g = 9.8 m s^{-2}.
Find the speed with which B hits the ground.

$a = (12 - 4)g \div (12 + 4) = 4.9$
$v^2 = 0 + 2 \times 4.9 \times 1.25 = 12.25$
$v = 3.5$
Answer: 3.5 m s^{-1}