Day 365 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the ages, a years, of 9 members of a club gives $\Sigma a = 237$ and $\Sigma a^2 = 7181$.
Find the mean and the standard deviation (3 s.f.).

Mean = $237 \div 9 = 26.3$

Variance = $\frac{7181}{9} - (26.333)^2 = 104.4$

Standard deviation = $\sqrt{104.4} = 10.2$

Answer: mean 26.3, standard deviation 10.2

2 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = x \ln x - 3x$.
Find the equation of the tangent to C at the point where $x = 1$, in the form $y = mx + c$.

$\frac{dy}{dx} = \ln x - 2$

At $x = 1$: $y = -3$ and $\frac{dy}{dx} = -2$

$y + 3 = -2(x - 1)$, so $y = -2x - 1$

Answer: $y = -2x - 1$

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(4x^2)^3 \times 6x}{x}$.

$(4x^2)^3 = 64x^6$

Numerator = $64 \times 6x^7 = 384x^7$

Divide: $\frac{384}{1} = 384$ and $x^7 \div x = x^6$

Answer: $384x^6$

4 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 3t^2 - 4t + 3$ m s^{-1}.
Find the distance moved between $t = 0$ and $t = 3$. (The particle does not change direction.)

$s = \int_0^3 (3t^2 - 4t + 3)\, dt$

$= [\, t^3 - 2t^2 + 3t \,]$ from $t = 0$ to $t = 3$

$= 18 - 0$

Answer: 18 m

5 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{7} 5 \times 2^r$.

This is a geometric series with first term 10, common ratio 2 and 7 terms

$S = 10(2^7 - 1) = 1270$

Answer: 1270

6 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 24t - 3t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 24 - 6t$

Maximum speed when $a = 0$: $t = 24 \div 6 = 4$

$v = 24 \times 4 - 3 \times 4^2$

Answer: 48 m s^{-1} at t = 4 s

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7 Two independent binomials (All boards)

X ~ B(7, 0.4) and Y ~ B(10, 0.1) are independent.
Find P(X = 3 and Y ≤ 1), to 3 significant figures.

P(X = 3) = 0.2903
P(Y ≤ 1) = 0.7361
Independent, so multiply: 0.2903 × 0.7361 = 0.214
Answer: 0.214

8 Area under a curve (All boards)

The region R is bounded by the axes, the line x = ln 3 and the curve $y = 3e^{3x} + 3$. Find the exact area of R.

Area = $\int_0^{\ln 3} (3e^{3x} + 3)\,dx = [3x + e^{3x}]_0^{\ln 3}$
= ln 27 + 26
Answer: ln 27 + 26

9 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-9x - 18}{(x + 1)(x + 4)}$ in partial fractions.

$\frac{-9x - 18}{(x + 1)(x + 4)} \equiv \frac{A}{(x + 1)} + \frac{B}{(x + 4)}$, so $-9x - 18 \equiv A(x + 4) + B(x + 1)$
x = −1: −9 = A × 3, so A = −3
x = −4: 18 = B × (−3), so B = −6
Answer: $\mathbf{-\frac{3}{(x + 1)} - \frac{6}{(x + 4)}}$

10 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + xy + 2x + y^2 - y = 3$.
Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at (−3, 0).

Differentiate with respect to x: $x\frac{dy}{dx} + 2x + 2y\frac{dy}{dx} + y + 2 - \frac{dy}{dx} = 0$
$\frac{dy}{dx} = \frac{-2x - y - 2}{x + 2y - 1}$
At (−3, 0): gradient = −1
Answer: $\mathbf{\frac{dy}{dx} = \frac{-2x - y - 2}{x + 2y - 1}}$**; gradient −1**