Day 364 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Normal distribution probabilities (All boards)

A battery lasts X hours, normal with mean 20, s.d. 6.
Find, to 3 s.f., the probability a value is
(a) more than 26,
(b) between 26 and 29.

$X \sim N(20, 6^2)$
(a) $P(X > 26) = 0.159$
(b) $P(26 < X < 29) = 0.9332 - 0.8413 = 0.0918$
Answer: (a) 0.159 (b) 0.0918

2 Solving exponential equations using logarithms (All boards)

Solve $1.5^x = 19$, giving your answer to 3 s.f.

Take logs: $x \ln 1.5 = \ln 19$
$x = \frac{\ln 19}{\ln 1.5} = 7.262$
Answer: $x = 7.26$

3 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of $y = f(x)$ onto the graph of:
(a) $y = f(x - 3)$
(b) $y = f(x) + 1$
(c) $y = f(\frac{x}{4})$

(a) $y = f(x - 3)$ is a translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.
(b) $y = f(x) + 1$ is a translation $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.
(c) $y = f(\frac{x}{4})$ is a stretch, scale factor 4, parallel to the x-axis.
Answer: (a) translation $\begin{pmatrix} \mathbf{3} \\ \mathbf{0} \end{pmatrix}$ (b) translation $\begin{pmatrix} \mathbf{0} \\ \mathbf{1} \end{pmatrix}$ (c) stretch, scale factor 4, parallel to the x-axis

4 Quotient rule (All boards)

Given that $y = \frac{5x^2 + 3}{x + 5}$,

find $\frac{dy}{dx}$, and its value when $x = 1$.

Quotient rule with $u = 5x^2 + 3$, $v = x + 5$
$u' = 10x$, $v' = 1$
$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{10x(x + 5) - 5x^2 - 3}{(x + 5)^2}$
When $x = 1$: $\frac{dy}{dx} = \frac{13}{9}$
Answer: $\frac{\mathbf{dy}}{\mathbf{dx}} = \frac{\mathbf{10x(x + 5) - 5x^2 - 3}}{\mathbf{(x + 5)^2}}$; value $\frac{\mathbf{13}}{\mathbf{9}}$

5 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A radio-controlled car moves with constant velocity $(2\mathbf{i} - 2\mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(8\mathbf{i} + 8\mathbf{j})$ m.
Find (a) its position vector when t = 5, (b) the value of t when it crosses the line through O in the direction of $\mathbf{i}$.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (8\mathbf{i} + 8\mathbf{j}) + 5(2\mathbf{i} - 2\mathbf{j}) = (18\mathbf{i} - 2\mathbf{j})$
(b) $\mathbf{j}$ component zero: $8 - 2t = 0$, so $t = 4$
Answer: (a) $(18\mathbf{i} - 2\mathbf{j})$ m (b) t = 4 s

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6 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

$P(A) = 0.3 + 0.2 = 0.5$ and $P(A \cap B) = 0.2$
Independent: $P(A \cap B) = P(A)P(B)$, so $P(B) = 0.2 \div 0.5 = 0.4$
$x = P(B) - P(A \cap B) = 0.4 - 0.2 = 0.2$
$y = 1 - (0.3 + 0.2 + 0.2) = 0.3$
Answer: x = 0.2, y = 0.3

7 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} -4 \\ 3 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a} \cdot \mathbf{b} = (-4)(-4) + (3)(3) + (4)(1) = 29$
$|\mathbf{a}| = \sqrt{41}$, $|\mathbf{b}| = \sqrt{26}$
$\cos\theta = \frac{29}{\sqrt{41}\sqrt{26}} = 0.8882$
$\theta = 27.3°$
Answer: 27.3°

8 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of
$(1 + 4x)(1 + 2x)^5$.

$(1 + 2x)^5$ has x^2 coefficient 40 and x^3 coefficient 80
x^3 term of the product $= 1 \times 80 + 4 \times 40$
$= 240$
Answer: 240

9 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 10 kg lies on a rough horizontal table (coefficient of friction 0.3). It is joined by a light string over a smooth pulley to a particle of mass 4 kg hanging freely.
Take $g = 9.8$ m s^{-2}.
The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.3 \times 10g = 10a$
Hanging: $4g - T = 4a$
Add: $4g - 3g = 14a$
$a = 0.7$
$T = 4(9.8 - 0.7)$
Answer: a = 0.7 m s^{-2}; T = 36.4 N

10 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A salary starts at £25000 in year 1 and increases by 4% each year.
Find the total salary earned in the first 7 years, to the nearest pound.

Geometric series with $a = 25000$, $r = 1.04$
$S_7 = \frac{25000(1.04^7 - 1)}{0.04}$
= £197,457
Answer: £197,457