Day 363 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = \sqrt{2x - 3}$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = 6$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = \frac{1}{\sqrt{2x - 3}}$

When $x = 6$: gradient $= \frac{1}{3}$

Answer: $\frac{dy}{dx} = \frac{1}{\sqrt{2x - 3}}$; **gradient** $\frac{1}{3}$

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-5 < 4x - 3 < 7$.

Subtract -3 from each part: $-2 < 4x < 10$

Divide each part by 4: $-\frac{1}{2} < x < \frac{5}{2}$

Answer: $-\frac{1}{2} < x < \frac{5}{2}$

3 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(6\mathbf{i} - 8\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north.
Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed $= \sqrt{(6^2 + -8^2)} = 10$
$\tan\theta = 8 \div 6$
Answer: 10 m s^{-1}; 53.1°

4 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$.

The point P lies on AB with AP : PB = 2 : 1.
Find the position vector of P.

$\text{AB} = \begin{pmatrix} 9 \\ -6 \end{pmatrix}$

$\text{AP} = \frac{2}{3}\text{AB} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{3} \\ \mathbf{-1} \end{pmatrix}$

5 Two independent binomials (All boards)

X ~ B(8, 0.2) and Y ~ B(9, 0.25) are independent.
Find P(X = 3 and Y ≤ 2), to 3 significant figures.

P(X = 3) = 0.1468
P(Y ≤ 2) = 0.6007
Independent, so multiply: 0.1468 × 0.6007 = 0.0882
Answer: 0.0882

Day 363 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^3 + n + 4$ is even for all integers n.

$n^3 + n = n(n^2 + 1)$

If n is even, $n(n^2 + 1)$ is even.

If n is odd, n^2 is odd and $n^2 + 1$ is even, so $n(n^2 + 1)$ is even.

So $n^3 + n$ is always even, and adding 4 gives an even number.

Answer: expression is even

7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-4x - 15}{(x + 3)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^2 \frac{-4x - 15}{(x + 3)(x + 4)}\,dx$,

giving the answer as a single logarithm.

(a) $\frac{-4x - 15}{(x + 3)(x + 4)} \equiv \frac{A}{x + 3} + \frac{B}{x + 4}$, so $-4x - 15 \equiv A(x + 4) + B(x + 3)$

$x = -3$: $A = -3$; $x = -4$: $B = -1$

(b) $\int_0^2 \left(\frac{-3}{x + 3} + \frac{-1}{x + 4}\right) dx = [-3\ln(x + 3) - \ln(x + 4)]_0^2$

$= -3\ln\frac{5}{3} - \ln\frac{6}{4} = \ln\left(\frac{18}{125}\right)$

Answer: (a) A = −3, B = −1 (b) $\ln\left(\frac{18}{125}\right)$

8 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg is held on a rough plane ($\tan\alpha = \frac{4}{3}$, $\mu = 0.4$) by a force P N up the plane, as shown.

Take $g = 9.8$ m s^{-2}. The block is about to move up the plane. Find P.

$R = mg\cos\alpha = 49 \times 3/5 = 29.4$

Friction (limiting) $F = \mu R = 11.76$

Moving up, friction acts down: $P = mg\sin\alpha + F$

$mg\sin\alpha = 39.2$

Answer: 50.96 N

9 Binomial: smallest n (All boards)

Given that each lottery ticket wins a prize independently with probability 0.3, find the least number of tickets needed for the probability of at least one success to exceed 0.95.

P(at least one) $= 1 - 0.7^n > 0.95$

$0.7^n < 0.05$

$n > \frac{\log 0.05}{\log 0.7} = 8.399$

Answer: n = 9

10 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x - 2)(x - 5)(x - 1) \geq 0$.

Critical values: $x = 1$, $x = 2$, $x = 5$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.

Reading from the sketch: $1 \leq x \leq 2$ or $x \geq 5$

Answer: $1 \leq x \leq 2$ or $x \geq 5$