Day 362 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown horizontally at 19 m s^{-1} from the top of a building 4.9 m above level ground.
Take g = 9.8 m s^{-2}.
Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $4.9 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 1$
t = 1 s
Horizontal: x = 19 × 1
Answer: 1 s, 19 m

2 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^3 - x^2 - 2x - 1$.
C is reflected in the y-axis.
Find its new equation.

Reflection in the y-axis: replace x by −x.
When reflected in the y-axis: $y = -x^3 - x^2 + 2x - 1$
Answer: $y = -x^3 - x^2 + 2x - 1$

3 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 5 strips to estimate

$\int_0^1 \cos(x^2)\,dx$,

giving your answer to 3 significant figures.

$h = \frac{1 - 0}{5} = 0.2$
x: 0, 0.2, 0.4, 0.6, 0.8, 1
y: 1.000, 0.9992, 0.9872, 0.9359, 0.8021, 0.5403
Area $\approx \frac{h}{2}[y_0 + y_5 + 2(y_1 + \ldots + y_4)]$
$= \frac{0.2}{2} \times [1.000 + 0.5403 + 2 \times 3.724] = 0.8989$
Answer: 0.899

4 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: mean pressure (hPa) at Camborne has mean 1011, sd 5.
Outliers are more than 2 sd from the mean.
Are the values 1020 and 999 outliers?

Limits: 1011 ± 2 × 5, so from 1001 to 1021
1020 is inside the limits, so it is not an outlier
999 is outside the limits, so it is an outlier
Answer: 1020: not an outlier; 999: outlier

5 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 5 s, (b) the distance between the stations.

(a) a = gradient = 15 ÷ 5
(b) Distance = area of trapezium = $\frac{1}{2} \times (25 + 10) \times 15$
Answer: (a) 3 m s^{-2} (b) 262.5 m

6 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 12x - 2y + 29 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:
$(x - 6)^2 - 36 + (y - 1)^2 - 1 + 29 = 0$
$(x - 6)^2 + (y - 1)^2 = 8$
Centre (6, 1), radius $2\sqrt{2}$
Answer: centre (6, 1), radius $2\sqrt{2}$

7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{2x + 8}{(x + 3)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{2x + 8}{(x + 3)(x + 5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{2x + 8}{(x + 3)(x + 5)} \equiv \frac{A}{x + 3} + \frac{B}{x + 5}$, so $2x + 8 \equiv A(x + 5) + B(x + 3)$
$x = -3$: $A = 1$; $x = -5$: $B = 1$
(b) $\int_0^4 (\frac{1}{x + 3} + \frac{1}{x + 5})\,dx = [\ln(x + 3) + \ln(x + 5)]_0^4$
$= \ln \frac{7}{3} + \ln \frac{9}{5} = \ln(\frac{21}{5})$

Answer: (a) A = 1, B = 1 (b) $\ln(\frac{21}{5})$

8 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ab^x$, where a and b are constants.
A plot of $\log_{10} y$ against x is a straight line with intercept 0.602 and gradient 0.699.
Find a and b to 2 significant figures.

$\log_{10} y = \log_{10} a + x \log_{10} b$
$\log_{10} a = 0.602$, so $a = 10^{0.602} = 4.0$
$\log_{10} b = 0.699$, so $b = 10^{0.699} = 5.0$
Answer: a = 4.0, b = 5.0

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-2 + 3\lambda = 0 + 1\mu$
j: $1 + 2\lambda = 5 + (-2)\mu$
Solving i and j: $\lambda = 1$, $\mu = 1$
k check: $-2 + 3(1) = 1$ and $1 + 0(1) = 1$, equal
Point: $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$
Answer: $\begin{pmatrix} \mathbf{1} \\ \mathbf{3} \\ \mathbf{1} \end{pmatrix}$

10 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = x + 8$ meets the circle $(x + 2)^2 + (y + 1)^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = x + 8$: $(x + 2)^2 + (x + 9)^2 = 25$

$x^2 + 11x + 30 = 0$

$(x + 5)(x + 6) = 0$, so $x = -6$ or $x = -5$

$y = x + 8$: A = $(-6, 2)$, B = $(-5, 3)$

$AB^2 = 1^2 + 1^2 = 2$

Answer: A$(-6, 2)$, B$(-5, 3)$, AB = $\sqrt{2}$