Day 361 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Locating roots: change of sign (All boards)

The function f(x) = $x^3 - 1x - 12$.
Show that f(x) = 0 has a root between x = 2 and x = 2.5.

f(2) = −6.00
f(2.5) = 1.12
f is continuous and changes sign, so there is a root in the interval
Answer: f(2) < 0 and f(2.5) > 0: sign change

2 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (−7, 0), (−3, −2) and (−1, 2).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB = $\frac{2}{-4} = -\frac{1}{2}$

Gradient BC = $\frac{4}{2} = 2$

$(-\frac{1}{2}) \times 2 = -1$, so AB and BC are perpendicular.

(b) AB = $2\sqrt{5}$ and BC = $2\sqrt{5}$

Area = $\frac{1}{2} \times$ AB × BC = 10

Answer: (b) area = 10

3 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the marks in a mock exam for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Class A: 45 − 35 = 10. Class B: 55 − 30 = 25
(b) Medians: 40 (Class A) and 45 (Class B), so Class B has the higher median
IQRs: Class B has the larger IQR, so its scores are more spread out
Answer: (a) 10 and 25 (b) Class B has the higher median; Class B is more spread out (larger IQR), Class A more consistent

4 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces (−3**i** − 5**j**) N and (7**i** + 2**j**) N act on a particle of mass 5 kg.
Find the magnitude of the acceleration of the particle.

Resultant = (−3**i** − 5**j**) + (7**i** + 2**j**) = 4**i** − 3**j**
|R| = $\sqrt{(4^2 + -3^2)} = 5$ N
a = F/m = 5 ÷ 5
Answer: 1 m s^{-2}

5 Binomial expected frequency (All boards)

A fair die is rolled 7 times and the number of sixes is X.
This is done 100 times. Estimate the number of times that X = 1.

X ~ B(7, $\frac{1}{6}$)

P(X = 1) = $^7C_1 \times (\frac{1}{6})^1 \times (\frac{5}{6})^6 = 0.3907$

Expected = 100 × 0.3907 = 39.07
Answer: about 39 times

6 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations

$2x + y = 10$

$x^2 + y^2 = 25$

$y = -2x + 10$; substitute: $x^2 + (-2x + 10)^2 = 25$

$5x^2 - 40x + 75 = 0$, so $x^2 - 8x + 15 = 0$

$(x - 5)(x - 3) = 0$, so $x = 5$ or $x = 3$

$y = 10 - 2x$ gives $y = 0$ and $y = 4$

Answer: $x = 3, y = 4$ and $x = 5, y = 0$

7 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2}$

to find the value of $\lim_{\theta \to 0} \frac{1 - \cos 4\theta}{\theta \sin\theta}$.

$1 - \cos 4\theta \approx 1 - (1 - \frac{4^2\theta^2}{2}) = \frac{16\theta^2}{2}$

$\theta \sin\theta \approx \theta \times \theta = \theta^2$

Ratio $= \frac{16\theta^2}{(2 \times 1\theta^2)} = 8$

Answer: 8

8 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 5 m and mass 80 kg rests horizontally on supports at C and D, where AC = 2 m and DB = 2 m.

A girl of mass 20 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero

Moments about D: $80\,g \times (3 - 2.5) = 20\,g \times x$

$80 \times 0.5 = 20\,x$

Answer: 2 m

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-1 + 2\lambda = 1 + 1\mu$

j: $-1 + 3\lambda = 2 + (-2)\mu$

Solving i and j: $\lambda = 1$, $\mu = 0$

k check: $0 + (-1)(1) = -1$ and $-1 + 3(0) = -1$, equal

Point: $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{-1} \end{pmatrix}$

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10 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $2^{2x} - 5\times2^{x} + 4 = 0$.

Let $y = 2^x$. Then $2^{2x} = y^2$, so $y^2 - 5y + 4 = 0$
$(y - 4)(y - 1) = 0$
$y = 4$ or $y = 1$
$2^x = 4$ gives $x = 2$; $2^x = 1$ gives $x = 0$.
Answer: $x = 0$ or $x = 2$