Day 36 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A puck of mass 2 kg is moving on a smooth horizontal surface at 8 m s^{-1} when a constant horizontal force of 20 N, in the direction of motion, acts for 6 s.
Find the speed of the puck at the end of this time and the distance it moves.

F = ma: a = 20 ÷ 2 = 10
v = u + at = 8 + 10 × 6
$s = ut + \frac{1}{2}at^2$
Answer: 68 m s^{-1}; 228 m

2 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic series is 3 and the common difference is 4.
The sum of the first n terms is 171. Find the value of n.

$S_n = \frac{n}{2}(2 \times 3 + (n - 1) \times 4) = 171$

$4n^2 + 2n - 342 = 0$
$2(n - 9)(2n + 19) = 0$
n must be positive, so n = 9
Answer: n = 9

3 Inequalities with exponents and logs (All boards)

Solve $5^x > 99$, giving your answer to 3 s.f.

Take logs: x ln 5 > ln 99
$x > \frac{\ln 99}{\ln 5} = 2.855$
Answer: x > 2.86

4 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 3 and Var(X) = 3.
Find (a) E(−2X − 1),
(b) Var(−2X − 1), (c) $E(X^2)$.

(a) E(aX + b) = aE(X) + b = (−2) × 3 − 1 = −7
(b) $Var(aX + b) = a^2Var(X) = 4 \times 3 = 12$
(c) $Var(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 3 + 9 = 12$
Answer: (a) −7 (b) 12 (c) 12

5 Particle in equilibrium: string and horizontal force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 1 kg hangs in equilibrium on a light string inclined at α to the vertical, held by a horizontal force P N, as shown. $\tan \alpha = \frac{12}{5}$.

Take g = 9.8 m s^{-2}.
Find the tension in the string and the value of P.

cos α = 5/13, sin α = 12/13
Vertical: T cos α = mg = 9.8, so T = 9.8 × 13/5
Horizontal: P = T sin α = 25.48 × 12/13
Answer: T = 25.48 N; P = 23.52 N

6 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:
(a) y = f(x − 5) (b) y = −f(x)

(a) y = f(x − 5): A(−1, −1) maps to (4, −1), B(1, −4) maps to (6, −4)
(b) y = −f(x): A(−1, −1) maps to (−1, 1), B(1, −4) maps to (1, 4)
Answer: (a) (4, −1), (6, −4) (b) (−1, 1), (1, 4)

7 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ab^x$, where a and b are constants.
A plot of $\log_{10} y$ against x is a straight line with intercept 0.699 and gradient 0.602.
Find a and b to 2 significant figures.

$\log_{10} y = \log_{10} a + x \log_{10} b$

$\log_{10} a = 0.699$, so $a = 10^{0.699} = 5.0$

$\log_{10} b = 0.602$, so $b = 10^{0.602} = 4.0$

Answer: a = 5.0, b = 4.0

8 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^3 - 3x^2 - 2x - 4$.
C is reflected in the x-axis.
Find its new equation.

Reflection in the x-axis: replace y by $-y$.

When reflected in the x-axis: $y = -x^3 + 3x^2 + 2x + 4$

Answer: $y = -x^3 + 3x^2 + 2x + 4$

9 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = 5xy$, given that $y = 4$ when $x = 0$.

Give y in terms of x.

$\frac{1}{y} dy = 5x\, dx$, so $\ln y = \frac{5x^2}{2} + c$

$x = 0, y = 4$: $c = \ln 4$

$y = 4\, e^{\frac{5}{2}x^2}$

Answer: $y = 4e^{\frac{5x^2}{2}}$

10 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = x^2 + 2$, the x-axis
and the lines $x = 0$ and $x = 2$.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^2 y^2\, dx = \pi \int_0^2 (x^4 + 4x^2 + 4)\, dx$

$= \pi\left[\frac{x^5}{5} + \frac{4x^3}{3} + 4x\right]_0^2$

$= \frac{376\pi}{15}$

Answer: $\frac{376\pi}{15}$