Day 359 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 12 pairs of data, $S_{xx} = 80$, $S_{yy} = 125$ and $S_{xy} = 63$.
Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{63}{\sqrt{80 \times 125}}$

$= \frac{63}{100.00} = 0.630$

r is positive, so there is weak positive correlation

Answer: r = 0.630, weak positive correlation

2 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (6e^{-2x} + \frac{2}{x})\,dx$.

$\int (6e^{-2x} + \frac{2}{x})\,dx = 2\ln x - 3e^{-2x}$

$= [2\ln x - 3e^{-2x}]_1^2$

$= \frac{3}{e^2} + 2\ln 2 - \frac{3}{e^4}$

Answer: $\frac{3}{e^2} + 2\ln 2 - \frac{3}{e^4}$

3 Poisson distribution (CIE)

X ~ Po(4.5), modelling the number of flaws per metre of cloth.
Find, to 3 significant figures,
(a) $P(X \leq 6)$
(b) $P(X \geq 6)$

(a) $P(X \leq 6) = P(X = 0) + ... + P(X = 6) = 0.831$
(b) $P(X \geq 6) = 1 - P(X \leq 5) = 0.297$

Answer: (a) 0.831 (b) 0.297

4 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^2 - 8x + 16$ for $x \geq 2$.
Find $f^{-1}(x)$ and state its domain.

Complete the square: $f(x) = 2(x - 2)^2 + 8$

$y = 2(x - 2)^2 + 8$, so $(x - 2)^2 = \frac{y - 8}{2}$

As $x - 2 \geq 0$ take the positive root: $x = 2 + \sqrt{\frac{y - 8}{2}}$

As $x - 2 \geq 0$, take the positive root: $x = 2 + \sqrt{\frac{y - 8}{2}}$

Answer: $f^{-1}(x) = 2 + \sqrt{\frac{x - 8}{2}}$, domain $x \geq 8$

5 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 70 kg stands on weighing scales in a lift. The lift decelerates at 0.5 m s^{-2} while moving upwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is downwards, magnitude 0.5
Taking up as positive: $R - 70 \times 9.8 = -70 \times 0.5$
$R = 70(9.8 - 0.5)$

Answer: 651 N

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6 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}$ and B has position vector $\begin{pmatrix} -11 \\ 3 \\ 1 \end{pmatrix}$.

The point P lies on AB with AP : PB = 1 : 3.

Find the position vector of P.

$\text{AB} = \begin{pmatrix} -12 \\ 8 \\ 4 \end{pmatrix}$

$\text{AP} = \frac{1}{4}\text{AB} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} -2 \\ -3 \\ -2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-2} \\ \mathbf{-3} \\ \mathbf{-2} \end{pmatrix}$

7 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 - 4x - 2$.

C is stretched by scale factor 3 parallel to the y-axis.

Find its new equation.

Stretch parallel to the y-axis: multiply y by the scale factor.

When stretched by scale factor 3 parallel to the y-axis: $y = 3x^2 - 12x - 6$

Answer: $y = 3x^2 - 12x - 6$

8 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 2 kg and 6 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.

Take $g = 9.8$ m s^{-2}.

Find the acceleration and the tension in the string.

Heavier: $6g - T = 6a$

Lighter: $T - 2g = 2a$

Add: $4g = 8a$, so $a = 4.9$

$T = 2(9.8 + 4.9)$

Answer: $a = 4.9$ m s^{-2}; $T = 29.4$ N

9 Rates of change (connected rates) (All boards)

The area of a square increases at a constant rate of 12 cm^{2}s^{-1}.

Find the rate of increase of the side length when the side is 5 cm.

$A = x^2$, so $\frac{dA}{dt} = 2x\frac{dx}{dt}$

$12 = 2 \times 5 \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{6}{5}$

Answer: $\frac{6}{5}$ cm/s

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10 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 3x^3 + px^2 + qx - 3$.

When $f(x)$ is divided by $(x - 3)$ the remainder is 60, and when divided by $(x - 2)$ the remainder is 11. Find p and q.

$f(3) = 60$: $81 + 9p + 3q - 3 = 60$, so $9p + 3q = -18$

$f(2) = 11$: $24 + 4p + 2q - 3 = 11$, so $4p + 2q = -10$

Solving simultaneously: $p = -1$, $q = -3$

Answer: $p = -1$, $q = -3$