Day 358 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The shaded region is the segment cut off by the chord AB of a circle with centre O and radius 9 cm. Angle AOB = 1.8 radians.
Find the area of the segment, to 3 s.f.

Sector area = $\frac{1}{2} \times 9^2 \times 1.8 = 72.9$

Triangle area = $\frac{1}{2} \times 9^2 \times \sin 1.8 = 39.44$

Segment = sector − triangle = 33.46 cm^2

Answer: 33.5 cm^2

2 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$4x - 5 \leq 6$ and $x^2 - 9x + 14 > 0$.

$4x - 5 \leq 6$ gives $x \leq \frac{11}{4}$

$x^2 - 9x + 14 > 0$: $(x - 2)(x - 7) > 0$ gives $x < 2$ or $x > 7$
Both must hold: $x < 2$
Answer: $x < 2$

3 Conditional probability from a count (All boards)

80 students: 35 play football, 21 play tennis, 39 do neither.
(a) How many do both?
(b) Given a person who plays football, find P(this person plays tennis).

(a) At least one: 80 − 39 = 41. Both = 35 + 21 − 41 = 15

(b) Of the 35 who play football, 15 also do the other: $P = \frac{15}{35} = \frac{3}{7}$

Answer: (a) 15 (b) $\frac{3}{7}$

4 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $(2\mathbf{i} + 8\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(-\mathbf{i} + 4\mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

$\mathbf{F} = m\mathbf{a}$: $(2\mathbf{i} + 8\mathbf{j}) + \mathbf{P} = 3(-\mathbf{i} + 4\mathbf{j}) = (-3\mathbf{i} + 12\mathbf{j})$
$\mathbf{P} = -5\mathbf{i} + 4\mathbf{j}$
$|\mathbf{P}| = \sqrt{(-5^2 + 4^2)}$
Answer: $\mathbf{P} = (-5\mathbf{i} + 4\mathbf{j})$ N; $|\mathbf{P}| = 6.40$ N

5 Two independent binomials (All boards)

$X \sim B(9, 0.2)$ and $Y \sim B(10, 0.6)$ are independent.
Find $P(X = 1 \text{ and } Y \leq 7)$, to 3 significant figures.

$P(X = 1) = 0.3020$
$P(Y \leq 7) = 0.8327$
Independent, so multiply: $0.3020 \times 0.8327 = 0.251$
Answer: 0.251

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6 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^3 \frac{3}{2x+1}\,dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{3}{2x+1}\,dx = \frac{3\ln(2x+1)}{2}$

$= \left[\frac{3\ln(2x+1)}{2}\right]_0^3$

$= \frac{3\ln 7}{2}$

Answer: $\mathbf{\frac{3\ln 7}{2}}$

7 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $x^2 - 6x + 17 > 0$ for all real values of x.

Complete the square: $x^2 - 6x + 17 = (x-3)^2 + 8$

$(x-3)^2 \geq 0$ for all real x, so $(x-3)^2 \geq 0$.

Adding 8: the expression is ≥ 8, so > 0 for all real x.

Answer: $\mathbf{x^2 - 6x + 17 > 0}$ **for all x**

8 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 3 \\ 8 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $3 + 3\lambda = 0 + 3\mu$

j: $8 + 3\lambda = 2 + 0\mu$

Solving i and j: $\lambda = -2$, $\mu = -1$

k check: $-3 + 0(-2) = -3$ and $-3 + 0(-1) = -3$, equal

Point: $\begin{pmatrix} -3 \\ 2 \\ -3 \end{pmatrix}$

Answer: $\mathbf{\begin{pmatrix} -3 \\ 2 \\ -3 \end{pmatrix}}$

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{5}$ is irrational, prove by contradiction that $7 + 3\sqrt{5}$ is irrational.

Assume $7 + 3\sqrt{5} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $3\sqrt{5} = \frac{r}{s} - 7 = \frac{r - 7s}{s}$, so $\sqrt{5} = \frac{r - 7s}{3s}$.

The right-hand side is a ratio of integers, so it is rational.

This contradicts $\sqrt{5}$ being irrational, so $7 + 3\sqrt{5}$ is irrational.

Answer: $\mathbf{7 + 3\sqrt{5}}$ **is irrational**

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10 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards at 4.9 m s^{-1} from a point 9.8 m above horizontal ground.
Take g = 9.8 m s^{-2}.
Find the time before it hits the ground.

Taking up as positive: $-9.8 = 4.9\,t - 4.9t^2$
$4.9t^2 - 4.9\,t - 9.8 = 0$
$t = (4.9 \pm \sqrt{216.09}) \div 9.8 = (4.9 \pm 14.7) \div 9.8$
Positive root only.
Answer: 2 s