Day 357 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 168; systematic sample of size 21.
(a) Find the sampling interval.
(b) The first item is number 1. Find the 3-th item.

(a) Interval = 168 ÷ 21 = 8
(b) 3-th item = 1 + 2 × 8 = 17
Answer: interval 8; item number 17

2 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 50 kg stands on weighing scales in a lift. The lift decelerates at 0.75 m s^{-2} while moving downwards. Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is upwards, magnitude 0.75
Taking up as positive: R − 50 × 9.8 = 50 × 0.75
R = 50(9.8 + 0.75)
Answer: 527.5 N

3 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^1 xe^{-x}\,dx$.

Let u = x and $\frac{dv}{dx} = e^{-x}$, so $\frac{du}{dx} = 1$ and $v = -e^{-x}$

Integral = $[uv] - \int v\frac{du}{dx}\,dx$

$= [-xe^{-x}]_0^1 - \int_0^1 -e^{-x}\,dx$

$= 1 - \frac{2}{e}$

Answer: $1 - \frac{2}{e}$

4 Poisson: changing the interval (CIE)

Faults occur at random in a cable at an average rate of 2 per hour of operation.
Find the probability that in 30 minutes there are exactly 0 (3 s.f.).

Rate in 30 minutes: $\lambda = 2 \times \frac{30}{60} = 1$

X ~ Po(1)

$P(X = 0) = e^{-1} \times \frac{1^0}{0!} = 0.368$

Answer: 0.368

5 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 4 kg is acted on by forces (7**i** + 3**j**) N and **P** N only. Its acceleration is (−**j**) m s^{-2}.
Find **P**, and its magnitude to 3 significant figures.

F = ma: (7**i** + 3**j**) + **P** = 4(−**j**) = (−4**j**)
P = −7**i** − 7**j**
|**P**| = $\sqrt{(-7^2 + -7^2)}$
Answer: P = (−7i − 7j) N; |P| = 9.90 N

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6 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}5\\4\\5\end{pmatrix}$ and B has position vector $\begin{pmatrix}0\\-6\\0\end{pmatrix}$.

The point P lies on AB with AP : PB = 2 : 3.

Find the position vector of P.

$\text{AB} = \begin{pmatrix}-5\\-10\\-5\end{pmatrix}$

$\text{AP} = \frac{2}{5}\text{AB} = \begin{pmatrix}-2\\-4\\-2\end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix}3\\0\\3\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{3}\\\mathbf{0}\\\mathbf{3}\end{pmatrix}$

7 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

x-intercepts at $x = -2, -1$ and 4, so $y = k(x + 2)(x + 1)(x - 4)$

The curve passes through $(0, -8)$: $k \times 2 \times 1 \times (-4) = -8$

So k = 1

Answer: f(x) = (x + 2)(x + 1)(x − 4)

8 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{2x + 4}{(x + 3)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{2x + 4}{(x + 3)(x + 5)}\,dx$,

giving the answer as a single logarithm.

(a) $\frac{2x + 4}{(x + 3)(x + 5)} \equiv \frac{A}{x + 3} + \frac{B}{x + 5}$, so $2x + 4 \equiv A(x + 5) + B(x + 3)$

$x = -3$: $A = -1$; $x = -5$: $B = 3$

(b) $\int_0^1 \left(\frac{-1}{x + 3} + \frac{3}{x + 5}\right) dx = \left[-\ln(x + 3) + 3\ln(x + 5)\right]_0^1$

$= -\ln\frac{4}{3} + 3\ln\frac{6}{5} = \ln\left(\frac{162}{125}\right)$

Answer: (a) A = −1, B = 3 (b) $\mathbf{\ln\left(\frac{162}{125}\right)}$

9 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 2)^2 + (y - 4)^2 = 20$.

The point P(−2, 2) lies on C.

Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (2, 4). Gradient of radius CP = $\frac{-2}{-4} = \frac{1}{2}$

The tangent is perpendicular to the radius: gradient = −2

$y - 2 = -2(x + 2)$

Multiply out and collect terms: $2x + y + 2 = 0$

Answer: 2x + y + 2 = 0

10 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers
m and n such that 4m + 6n = 147.

Assume there are integers m and n with 4m + 6n = 147.
4m + 6n = 2(2m + 3n).
So the left-hand side is a multiple of 2.
But 147 = 2 × 73 + 1 is not a multiple of 2.
This is a contradiction, so no such integers exist.
Answer: no integer solutions (LHS is a multiple of 2, 147 is not)