Day 356 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Standardising to compare (All boards)

Marks are normal: Biology N(70, 5^2), Chemistry N(40, 8^2).
Anna scores 72.5 in Biology and 36 in Chemistry. Which is her better performance, relatively?

z-score in Biology = $\frac{72.5 - 70}{5} = 0.500$

z-score in Chemistry = $\frac{36 - 40}{8} = -0.500$

The higher z-score is in Biology

Answer: Biology

2 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 5x < 6$.

Rearrange: $x^2 - 5x - 6 < 0$
Factorise: $(x - 6)(x + 1) < 0$
Critical values: $x = -1$ and $x = 6$
Sketch the ∪-shaped parabola: $-1 < x < 6$
Answer: $-1 < x < 6$

3 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moving in a straight line with constant acceleration increases its speed from 3 m s^{-1} to 15 m s^{-1} while covering 54 m.
Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $225 = 9 + 108\,a$, so $a = 2$
(b) $v = u + at$: $15 = 3 + 2t$
$t = 12 \div 2 = 6$
Answer: (a) 2 m s^{-2} (b) 6 s

4 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the trapezium rule with 4 strips.

Use it to estimate $\int_1^3 \frac{\ln x}{x}\,dx$, to 3 s.f.

$h = \frac{3 - 1}{4} = 0.5$

x: 1, 1.5, 2, 2.5, 3
y: 0, 0.2703, 0.3466, 0.3665, 0.3662

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2} \times [0 + 0.3662 + 2 \times 0.9834] = 0.5833$

Answer: 0.583

5 Transformations of a turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

y = f(x) has a maximum point at (−4, −4).
Find the turning point on the curve:
(a) y = f(−x) (b) y = 2f(x)

(c) $y = f(\frac{x}{2})$

(a) y = f(−x): the point (−4, −4) maps to (4, −4)
(b) y = 2f(x): the point (−4, −4) maps to (−4, −8)

(c) $y = f(\frac{x}{2})$: the point (−4, −4) maps to (−8, −4)

Answer: (a) (4, −4) (b) (−4, −8) (c) (−8, −4)

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6 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf f(x) = kx for $0 \le x \le 3$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 2). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^2/2]$ from 0 to 3 = $k \times \frac{3^2}{2} = 1$, so $k = \frac{2}{9}$

(b) $P(X > 2) = \int_2^3 \frac{2}{9} x\,dx = 1 - \frac{2^2}{3^2} = \frac{5}{9}$

(c) $E(X) = \int_0^3 x\,f(x)\,dx = \frac{2}{9} \times \frac{3^3}{3} = 2$

Answer: (a) $k = \frac{2}{9}$ (b) $\frac{5}{9}$ (c) 2

7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of
$(3 - 4x)(1 - 3x)^4$.

$(1 - 3x)^4$ has x^2 coefficient 54 and x^3 coefficient -108
x^3 term of the product = $3 \times (-108) + (-4) \times 54$
$= -540$
Answer: -540

8 Standard integrals (definite) (All boards)

Find the exact value of
$\int_1^4 (x^2 + \frac{2}{x})\,dx$.

$\int (x^2 + \frac{2}{x})\,dx = \frac{x^3}{3} + 2 \ln x$

$= [\frac{x^3}{3} + 2 \ln x]_1^4$

$= \ln 16 + 21$
Answer: ln 16 + 21

9 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 3 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{4}{3}$ and $\tan\beta = \frac{3}{4}$.

Take g = 9.8 m s^{-2}.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{3}{5} = T_2 \times \frac{4}{5}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 29.4$

From the first: $T_1 = \frac{4}{3} T_2$; into the second: $T_2 \times \frac{5}{3} = 29.4$

Answer: T_1 = 23.52 N (at α); T_2 = 17.64 N (at β)

10 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = (2 + y)e^{-x}$,

given that y = 1 when x = 0.

$\frac{1}{2+y}\,dy = e^{-x}\,dx$, so $\ln(2 + y) = -e^{-x} + c$

x = 0: $\ln 3 = -1 + c$, so $c = 1 + \ln 3$

$2 + y = 3\,e^{1 - e^{-x}}$, so $y = 3\,e^{1 - e^{-x}} - 2$

Answer: $y = 3e^{1 - e^{-x}} - 2$