Day 355 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = (3x^2 + 1)^3$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = -2$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = 18x(3x^2 + 1)^2$

When $x = -2$: gradient $= -6084$

Answer: $\frac{dy}{dx} = 18x(3x^2 + 1)^2$; gradient -6084

2 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(3\mathbf{i} + 6\mathbf{j})$ N and $(3\mathbf{i} + 2\mathbf{j})$ N act on a particle of mass 2 kg.
Find the magnitude of the acceleration of the particle.

Resultant $= (3\mathbf{i} + 6\mathbf{j}) + (3\mathbf{i} + 2\mathbf{j}) = 6\mathbf{i} + 8\mathbf{j}$

$|R| = \sqrt{(6^2 + 8^2)} = 10$ N

$a = F/m = 10 \div 2$

Answer: 5 m s^{-2}

3 Binomial expected frequency (All boards)

Each of 9 people independently supports a team with probability $\frac{1}{3}$; X is the number of supporters.

This is done 500 times. Estimate the number of times that X = 2.

$X \sim B(9, \frac{1}{3})$

$P(X = 2) = {}^9C_2 \times (\frac{1}{3})^2 \times (\frac{2}{3})^7 = 0.2341$

Expected $= 500 \times 0.2341 = 117.1$

Answer: about 117 times

4 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line with constant acceleration increases its speed from 7 m s^{-1} to 24.5 m s^{-1} while covering 110.25 m.
Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $600.25 = 49 + 220.5\,a$, so $a = 2.5$

(b) $v = u + at$: $24.5 = 7 + 2.5\,t$

$t = 17.5 \div 2.5 = 7$

Answer: (a) 2.5 m s^{-2} (b) 7 s

5 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $4x + 3y = 0$ and line l_2 has equation $2x - y = -10$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (−3, 4)

l_1 meets $y = 0$ at $x = 0$, and l_2 meets $y = 0$ at $x = -5$

Base = 5, height = 4

Area $= \frac{1}{2} \times$ base $\times$ height $= 10$

Answer: 10

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6 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{4x + 6}{(x + 1)(x + 3)}$ in partial fractions.

(b) Hence evaluate $\int_0^2 \frac{4x + 6}{(x + 1)(x + 3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{4x + 6}{(x + 1)(x + 3)} \equiv \frac{A}{x + 1} + \frac{B}{x + 3}$, so $4x + 6 \equiv A(x + 3) + B(x + 1)$
$x = -1$: $A = 1$; $x = -3$: $B = 3$

(b) $\int_0^2 (\frac{1}{x + 1} + \frac{3}{x + 3})\,dx = [\ln(x + 1) + 3\ln(x + 3)]_0^2$
$= \ln\frac{3}{1} + 3\ln\frac{5}{3} = \ln(\frac{125}{9})$

Answer: (a) A = 1, B = 3 (b) $\ln(\frac{125}{9})$

7 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has second term -27 and fifth term 1.
Find (a) the common ratio, (b) the first term, (c) the sum to infinity.

$ar = -27$ and $ar^4 = 1$, so $r^3 = \frac{1}{-27} = -\frac{1}{27}$

$r = -\frac{1}{3}$, and $a = -27 \div (-\frac{1}{3}) = 81$

$S_\infty = \frac{a}{1 - r} = \frac{81}{1 - (-\frac{1}{3})} = \frac{243}{4}$

Answer: (a) $r = -\frac{1}{3}$ (b) $a = 81$ (c) $\frac{243}{4}$

8 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = e^{2x}$, the x-axis
and the lines $x = 0$ and $x = 1$.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi\int_0^1 y^2\,dx = \pi\int_0^1 e^{4x}\,dx$

$= \pi[\frac{e^{4x}}{4}]_0^1$

$= -\frac{\pi(1 - e^4)}{4}$

Answer: $-\frac{\pi(1 - e^4)}{4}$

9 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 10 ml. Set mean 100 ml; 9 bottles have mean 106.9 ml.
Test at the 5% level whether the true mean is greater than 100.

H_0: $\mu = 100$, H_1: $\mu > 100$. Under H_0, $\overline{X} \sim N(100, \frac{10^2}{9})$

$z = \frac{106.9 - 100}{10/\sqrt{9}} = 2.070$
$P(Z > 2.070) = 0.0192 < 0.05$
Reject H_0: there is sufficient evidence that the mean has increased
Answer: reject H_0: sufficient evidence of an increase in the mean

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10 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Show that the equation $x^2 - 2x + 6 = 0$ has no real roots.

$b^2 - 4ac = (-2)^2 - 4 \times 1 \times 6$

$= 4 - 24 = -20$

The discriminant is negative, so there are no real roots.

Answer: $b^2 - 4ac = -20 < 0$