Day 354 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that y = cos 4x,

find $\frac{dy}{dx}$, and the gradient of the curve when x = $\frac{\pi}{12}$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = -4 \sin 4x$

When x = $\frac{\pi}{12}$: gradient = $-2\sqrt{3}$

Answer: $\frac{dy}{dx} = -4 \sin 4x$; gradient $-2\sqrt{3}$

2 Normal distribution probabilities (All boards)

The length X cm of a metal rod is normal with mean 30, s.d. 3.
Find, to 3 s.f., the probability a value is
(a) more than 34.5,
(b) between 34.5 and 39.

$X \sim N(30, 3^2)$
(a) $P(X > 34.5) = 0.0668$
(b) $P(34.5 < X < 39) = 0.9987 - 0.9332 = 0.0655$
Answer: (a) 0.0668 (b) 0.0655

3 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $(\frac{81}{16})^{\frac{1}{4}}$.

$\frac{81}{16} = (\frac{3}{2})^4$, so the expression = $(\frac{3}{2})^1$

$= \frac{3}{2}$

Answer: $\frac{3}{2}$

4 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (2e^{3x} + \frac{5}{x})\, dx$.

$\int (2e^{3x} + \frac{5}{x})\, dx = \frac{2e^{3x}}{3} + 5 \ln x$

$= [\frac{2e^{3x}}{3} + 5 \ln x]_1^2$

$= \ln 32 + \frac{2e^6}{3} - \frac{2e^3}{3}$

Answer: $\ln 32 + \frac{2e^6}{3} - \frac{2e^3}{3}$

5 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when
$2x^3 - x^2 + 2x + 9$ is divided by $(x + 1)$.

$2x^2 \times (x + 1) = 2x^3 + 2x^2$, leaving $-3x^2 + 2x + 9$
$-3x \times (x + 1) = -3x^2 - 3x$, leaving $5x + 9$
$5 \times (x + 1) = 5x + 5$, leaving 4
Quotient $2x^2 - 3x + 5$, remainder 4
Answer: quotient $2x^2 - 3x + 5$, remainder 4

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6 Simplify using double angle formulae (All boards)

Simplify $\frac{\cos 2x}{\cos x + \sin x}$,
giving your answer in its simplest form.

$\cos 2x = \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x)$
so the expression is $\cos x - \sin x$
Answer: $\cos x - \sin x$

7 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (9 kg) and B (11 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 2.25 m above the ground, then released.
Take $g = 9.8$ m s^{-2}.
Find the speed with which B hits the ground.

$a = (11 - 9)g \div (11 + 9) = 0.98$
$v^2 = 0 + 2 \times 0.98 \times 2.25 = 4.41$
$v = 2.1$
Answer: 2.1 m s^{-1}

8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 8 g. Claimed mean 200 g; 16 packets have mean 205.1 g.
Test at the 5% level whether the true mean is greater than 200.

H_0: $\mu = 200$, H_1: $\mu > 200$. Under H_0, $\overline{X} \sim N(200, \frac{8^2}{16})$

$z = \frac{205.1 - 200}{8/\sqrt{16}} = 2.550$
$P(Z > 2.550) = 0.0054 < 0.05$
Reject H_0: there is sufficient evidence that the mean has increased
Answer: reject H_0: sufficient evidence of an increase in the mean

9 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected from level ground with horizontal velocity component 23 m s^{-1} and vertical velocity component 44.1 m s^{-1} upwards.
Take $g = 9.8$ m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 44.1\,t - 4.9t^2$, so $t = 44.1 \div 4.9$
(b) Range = horizontal speed × time = 23×9
Answer: (a) 9 s (b) 207 m

10 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 3x + 4$ for $-3 < x \leq 2$.
Find the range of f.

f is increasing, so the extreme values occur at the ends of the domain.
$f(-3) = -5$ and $f(2) = 10$
$x = -3$ is excluded and $x = 2$ is included, so $f = -5$ is excluded and $f = 10$ is included.
Answer: $-5 < f(x) \leq 10$