Day 353 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + kx^2 - 8x + 24$.
Given that $(x - 3)$ is a factor of $f(x)$,
(a) find the value of k,
(b) factorise $f(x)$ completely.

$f(3) = 0$: $54 + 9k - 24 + 24 = 0$
$k = -6$
$f(x) = (x - 3)(2x^2 - 8)$
$= 2(x - 3)(x - 2)(x + 2)$
Answer: (a) $k = -6$ (b) $f(x) = 2(x - 3)(x - 2)(x + 2)$

2 Solving exponential equations using logarithms (All boards)

Solve $e^{-3x} = 10$, giving your answer to 3 s.f.

Take natural logs: $-3x = \ln 10$
$x = \frac{\ln 10}{-3} = -0.7675$
Answer: $x = -0.768$

3 Standardising to compare (All boards)

Marks are normal: Maths $N(48, 6^2)$, Physics $N(40, 8^2)$.
Anna scores 55.5 in Maths and 36 in Physics. Which is her better performance, relatively?

z-score in Maths $= \frac{55.5 - 48}{6} = 1.25$
z-score in Physics $= \frac{36 - 40}{8} = -0.500$
The higher z-score is in Maths
Answer: Maths

4 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, with $R > 0$ and $0 < \alpha < 90°$.
(b) Hence find its maximum value and the smallest $\theta \geq 0$ at which it occurs. Give α to 1 d.p.

$R = \sqrt{6^2 + 8^2} = 10$, $\tan\alpha = \frac{8}{6}$ so $\alpha = 53.1°$
Maximum is R when $\cos(\theta - \alpha) = 1$, i.e. $\theta = \alpha$
Answer: (a) $10\cos(\theta - 53.1°)$ (b) max 10 at $\theta = 53.1°$

5 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(2n + 3)^2 - (2n - 3)^2$ is a multiple of 24 for all integer values of n.

$(2n + 3)^2 = 4n^2 + 12n + 9$
$(2n - 3)^2 = 4n^2 - 12n + 9$
Difference $= 24n = 24 \times n$
n is an integer, so this is a multiple of 24 for every integer n.
Answer: difference = 24n, a multiple of 24

6 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{5}{13}$ and $\cos B = \frac{3}{5}$, where A and B are acute,
find the exact value of $\cos(A - B)$.

$\cos A = \frac{12}{13}$ and $\sin B = \frac{4}{5}$
$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$= \frac{12}{13} \times \frac{3}{5} + \frac{5}{13} \times \frac{4}{5} = \frac{56}{65}$
Answer: $\frac{56}{65}$

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7 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = 3t^3 + 4t^2$ metres.
Find (a) the average velocity over the first 6 seconds, (b) the velocity at t = 6.

(a) s(6) = 792, so average velocity = 792 ÷ 6

(b) $v = ds/dt = 9t^2 + 8t$, v(6) = 372

Answer: (a) 132 m s^{-1} (b) 372 m s^{-1}

8 Probability without replacement (All boards)

A bag has 4 white and 9 black sweets; two are taken without replacement.
(a) Find P(same colour).
(b) Given they are the same colour, find P(both white).

(a) P(white white) = $\frac{4}{13} \times \frac{3}{12} = \frac{1}{13}$

P(black black) = $\frac{9}{13} \times \frac{8}{12} = \frac{6}{13}$

P(same) = $\frac{1}{13} + \frac{6}{13} = \frac{7}{13}$

(b) P = $\frac{1}{13} \div \frac{7}{13} = \frac{1}{7}$

Answer: (a) $\frac{7}{13}$ (b) $\frac{1}{7}$

9 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -3x - 12$ meets the circle $x^2 + (y + 2)^2 = 50$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -3x - 12$: $x^2 + (-3x - 10)^2 = 50$

$x^2 + 6x + 5 = 0$

$(x + 1)(x + 5) = 0$, so $x = -5$ or $x = -1$

$y = -3x - 12$: A = (−5, 3), B = (−1, −9)

$AB^2 = 4^2 + (-12)^2 = 160$

Answer: A(−5, 3), B(−1, −9), AB = $4\sqrt{10}$

10 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 5 m and mass 30 kg rests horizontally on supports at C and D, where AC = 2 m and DB = 1.5 m.
A girl of mass 25 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero

Moments about D: 30 g × (3.5 − 2.5) = 25 g × x

30 × 1 = 25 x

Answer: 1.2 m