Day 352 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Union, conditional probability and independence (All boards)

P(A) = 0.4, P(B) = 0.6, P(A ∩ B) = 0.09.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) P(A ∪ B) = 0.4 + 0.6 − 0.09 = 0.91

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.09}{0.6} = 0.15$

(c) P(A)P(B) = 0.4 × 0.6 = 0.24, which does not equal P(A ∩ B)

Answer: (a) 0.91 (b) 0.15 (c) not independent

2 Basic trig equations (degrees) (All boards)

Solve $\cos x = -\frac{\sqrt{3}}{2}$ for $0 \le x \le 360°$.

Principal value x = 150°
Second solution: 360° − 150° = 210°

Answer: x = 150°, 210°

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "$n^2 + n$ is a multiple of 6 for every positive integer n."
Find a counter-example.

Try n = 1, 2, 3, ... until $n^2 + n$ is not a multiple of 6.
n = 1: $n^2 + n = 2$, and 2 ÷ 6 is not an integer.

Answer: n = 1 (value 2)

4 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int \frac{2}{4x + 6}\,dx$,

(b) the exact value of $\int_0^3 \frac{2}{4x + 6}\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{\ln(4x + 6)}{2} + c$

(b) $\left[\frac{\ln(4x + 6)}{2}\right]_0^3$

$= \frac{\ln 3}{2}$

Answer: (a) $\frac{\ln(4x + 6)}{2} + c$ (b) $\frac{\ln 3}{2}$

5 Work-energy: sliding to rest (CIE)

A block of mass 2 kg is projected along a rough horizontal surface with speed 2.8 m s^{-1} and comes to rest after sliding 2 m.
Take g = 9.8 m s^{-2}.
Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE
$\mu \times 2 \times 9.8 \times 2 = ½ \times 2 \times 2.8^2$
$39.2\,\mu = 7.84$

Answer: 0.2

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6 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x + 18$ meets the circle $(x - 2)^2 + (y - 3)^2 = 169$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -x + 18$: $(x - 2)^2 + (-x + 15)^2 = 169$
$x^2 - 17x + 30 = 0$
$(x - 15)(x - 2) = 0$, so $x = 2$ or $x = 15$
$y = -x + 18$: A = (2, 16), B = (15, 3)
$AB^2 = 13^2 + (-13)^2 = 338$
Answer: A(2, 16), B(15, 3), AB = $13\sqrt{2}$

7 Two independent binomials (All boards)

$X \sim B(12, 0.5)$ and $Y \sim B(11, 0.6)$ are independent.
Find $P(X = 5 \text{ and } Y \leq 6)$, to 3 significant figures.

$P(X = 5) = 0.1934$
$P(Y \leq 6) = 0.4672$
Independent, so multiply: $0.1934 \times 0.4672 = 0.0903$
Answer: 0.0903

8 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^4 (2x^3 + 1 - \frac{1}{x^2})\,dx$.

$\int (2x^3 + 1 - \frac{1}{x^2})\,dx = \frac{x^4}{2} + x + \frac{1}{x}$

$= [\frac{x^4}{2} + x + \frac{1}{x}]_1^4$

$= \frac{519}{4}$

Answer: $\frac{519}{4}$

9 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $2^{2x} + 2^x - 2 = 0$.

Let $y = 2^x$. Then $2^{2x} = y^2$, so $y^2 + y - 2 = 0$
$(y - 1)(y + 2) = 0$
$y = 1$ or $y = -2$
$2^x > 0$ for all x, so $y = -2$ is rejected. $2^x = 1$ gives $x = 0$.
Answer: x = 0

10 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A remote-control car moves with constant velocity $(-20\mathbf{i} - 21\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north. Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed $= \sqrt{(-20^2 + -21^2)} = 29$
$\tan\theta = 21 \div 20$
Answer: 29 m s^{-1}; 46.4°