Day 351 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train moves in a straight line with constant acceleration. Its initial speed is 13 m s^{-1}, its final speed is 17.5 m s^{-1} and it travels 45.75 m.
Find the time taken.

$s = ½(u + v)t$
$45.75 = ½ \times (13 + 17.5) \times t$
$t = 91.5 \div 30.5$
Answer: 3 s

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 319; systematic sample of size 29.
(a) Find the sampling interval.
(b) The first item is number 1. Find the 7-th item.

(a) Interval $= 319 \div 29 = 11$
(b) 7-th item $= 1 + 6 \times 11 = 67$
Answer: interval 11; item number 67

3 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 4x - 9$. Use the Newton-Raphson method with $x_0 = 2$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 3x^2 + 4$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 2 - \frac{7.000}{16.00} = 1.5625$$

$$x_2 = 1.5625 - \frac{1.065}{11.32} = 1.4685$$

Answer: $x_1 = 1.5625$, $x_2 = 1.4685$

4 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

$P(X = x) = k(5 - x)$ for $x = 1, 2, ..., 4$.
(a) Find k.
(b) Find E(X) and Var(X).

(a) $k(4 + 3 + 2 + 1) = 1$, so $10k = 1$ and $k = \frac{1}{10}$

(b) $E(X) = 1 \times \frac{2}{5} + 2 \times \frac{3}{10} + 3 \times \frac{1}{5} + 4 \times \frac{1}{10} = 2$

$E(X^2) = 5$, so $Var(X) = 5 - (2)^2 = 1$

Answer: (a) $k = \frac{1}{10}$ (b) E(X) = 2, Var(X) = 1

5 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (−2, 1), (−1, −1) and (1, 0).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB $= \frac{2}{-1} = -2$

Gradient BC $= \frac{1}{2}$

$(-2) \times \frac{1}{2} = -1$, so AB and BC are perpendicular.

(b) AB $= \sqrt{5}$ and BC $= \sqrt{5}$
Area $= \frac{1}{2} \times AB \times BC = \frac{5}{2}$

Answer: (b) area $= \frac{5}{2}$

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6 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\cos\theta = -\frac{21}{29}$ and $90° < \theta < 180°$,

find the exact value of $\sin 2\theta$.

$\sin\theta = \sqrt{1 - (-\frac{21}{29})^2} = \frac{20}{29}$

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{20}{29} \times (-\frac{21}{29}) = -\frac{840}{841}$

Answer: $-\frac{840}{841}$

7 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $2t^2 - 18t + 36$ m s^{-1}.
Find the first time P is at instantaneous rest, and its distance from O then.

$v = 0$: $2(t - 3)(t - 6) = 0$, so $t = 3$ or $t = 6$

First time $t = 3$

$s = \int_0^3 v\,dt = [\frac{2}{3}t^3 - 9t^2 + 36t]$ from 0 to 3

Answer: t = 3 s; 45 m

8 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The depth of water at a pier is modelled by $D = 8 + 6\sin(\frac{\pi t}{6})$, where t is in hours after noon.

Find (a) the greatest and least values, (b) the values of t with $0 \le t \le 12$ when it equals 5 m.

(a) sin ranges from -1 to 1: greatest $8 + 6 = 14$, least $8 - 6 = 2$

(b) $8 + 6\sin(\frac{\pi t}{6}) = 5$, so $\sin(\frac{\pi t}{6}) = -\frac{1}{2}$

$\frac{\pi t}{6} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, so $t = 7$ or 11

Answer: (a) 14 m, 2 m (b) t = 7 and 11

9 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + px^2 + qx - 6$.

When f(x) is divided by $(x - 3)$ the remainder is 78, and when divided by $(x - 2)$ the remainder is 32. Find p and q.

$f(3) = 78$: $27 + 9p + 3q - 6 = 78$, so $9p + 3q = 57$

$f(2) = 32$: $8 + 4p + 2q - 6 = 32$, so $4p + 2q = 30$

Solving simultaneously: $p = 4$, $q = 7$

Answer: p = 4, q = 7

10 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(4n + 5)^2 - (4n - 5)^2$ is a multiple of 80 for all integer values of n.

$(4n + 5)^2 = 16n^2 + 40n + 25$

$(4n - 5)^2 = 16n^2 - 40n + 25$

Difference $= 80n = 80 \times n$

n is an integer, so this is a multiple of 80 for every integer n.

Answer: difference = 80n, a multiple of 80