Day 350 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 5 cm and angle 30°.
Find, in terms of π, the arc length and the area of the sector.

$\theta = 30° = \frac{\pi}{6}$ radians

Arc length $= r\theta = \frac{5\pi}{6}$ cm

Area $= \frac{1}{2}r^2\theta = \frac{25\pi}{12}$ cm^2

Answer: arc $\frac{5\pi}{6}$ cm, area $\frac{25\pi}{12}$ cm^2

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-2 < -3x - 4 < 10$.

Subtract -4 from each part: $2 < -3x < 14$

Divide each part by -3 (reversing the inequalities): $-\frac{14}{3} < x < -\frac{2}{3}$

Answer: $-\frac{14}{3} < x < -\frac{2}{3}$

3 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 10 kg rests on a rough horizontal floor, with coefficient of friction 0.5. A horizontal force of 39 N is applied to the box.
Take g = 9.8 m s^{-2}.
Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 10 \times 9.8 = 98$, maximum friction $\mu R = 0.5 \times 98 = 49$
$39 < 49$ so the box stays at rest
Friction balances the applied force: $F = 39$
Answer: Does not move; friction = 39 N

4 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = \frac{24}{7}$ and $0 < \theta < 90°$,
find the exact value of $\sin 2\theta$.

Right-angled triangle 24, 7, 25: $\sin\theta = \frac{24}{25}$, $\cos\theta = \frac{7}{25}$

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{24}{25} \times \frac{7}{25} = \frac{336}{625}$

Answer: $\frac{336}{625}$

5 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (3, 2) and B is the point (4, 7).
Find the perpendicular bisector of AB as $ax + by + c = 0$, with integer a, b, c.

Midpoint of AB $= (\frac{7}{2}, \frac{9}{2})$

Gradient of AB $= \frac{5}{1} = 5$

Perpendicular gradient $= -\frac{1}{5}$

$y - \frac{9}{2} = -\frac{1}{5}(x - \frac{7}{2})$, which gives $x + 5y - 26 = 0$

Answer: $x + 5y - 26 = 0$

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6 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 6 kg is pulled across smooth horizontal ice by a rope with tension 30 N, inclined at α above the horizontal, where $\tan\alpha = \frac{3}{4}$.

Take $g = 9.8$ m s^{-2}.

Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 4/5$, $\sin\alpha = 3/5$

(a) Horizontal: $30 \times 4/5 = 6a$

(b) Vertical: $R + 30 \times 3/5 = 6 \times 9.8$

Answer: (a) 4 m s^{-2} (b) 40.8 N

7 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $7x + 8y = 42$ and line l_2 has equation $7x + 3y = 7$.

The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = $(-2, 7)$

l_1 meets $y = 0$ at $x = 6$, and l_2 meets $y = 0$ at $x = 1$

Base = 5, height = 7

Area = $\frac{1}{2} \times$ base $\times$ height = $\frac{35}{2}$

Answer: $\frac{35}{2}$

8 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, texts sent in a day by 11 students:

15, 16, 20, 20, 21, 24, 31, 36, 38, 40, 69.

Find Q_1, Q_3, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = 20$, $Q_3 = 38$, IQR = 18

Lower limit = $20 - 1.5 \times 18 = -7$

Upper limit = $38 + 1.5 \times 18 = 65$

Outlier(s): 69

Answer: $Q_1 = 20$, $Q_3 = 38$, IQR = 18; outlier 69

9 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The depth of water at a pier is modelled by $D = 8 + 4\sin(\frac{\pi t}{6})$, where t is in hours after noon.

Find (a) the greatest and least values, (b) the values of t with $0 \le t \le 12$ when it equals 6 m.

(a) $\sin$ ranges from -1 to 1: greatest $8 + 4 = 12$, least $8 - 4 = 4$

(b) $8 + 4\sin(\frac{\pi t}{6}) = 6$, so $\sin(\frac{\pi t}{6}) = -\frac{1}{2}$

$\frac{\pi t}{6} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, so $t = 7$ or 11

Answer: (a) 12 m, 4 m (b) t = 7 and 11

10 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: mean pressure (hPa) at Camborne has mean 1014, sd 5.

Outliers are more than 2 sd from the mean.

Are the values 1021 and 1006 outliers?

Limits: $1014 \pm 2 \times 5$, so from 1004 to 1024

1021 is inside the limits, so it is not an outlier

1006 is inside the limits, so it is not an outlier

Answer: 1021: not an outlier; 1006: not an outlier