Day 35 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Total probability and Bayes (All boards)

R: has the condition; L: test is positive.
P(R) = 0.6, P(L | R) = 0.4, P(L | R') = 0.25.
(a) Find P(L).
(b) Find P(R | L).

(a) $P(L) = 0.6 \times 0.4 + 0.4 \times 0.25 = 0.24 + 0.1 = 0.34$

(b) $P(R \mid L) = \frac{0.6 \times 0.4}{0.34} = \frac{0.24}{0.34} = 0.706$

Answer: (a) 0.34 (b) $\frac{12}{17}$

2 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 - 6x + 4$ for $x \leq 5$.
Find the range of f.

$f(x) = (x - 3)^2 - 5$, with minimum at $x = 3$
$x = 3$ is in the domain, so the minimum value is -5.
Answer: $f(x) \geq -5$

3 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sector OAB of a circle with centre O, radius 5 cm and angle 1.6 radians.
Find (a) the arc length AB, (b) the area of the sector.

Arc length $= r\theta = 5 \times 1.6 = 8$ cm

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times 1.6 = 20$ cm^2

Answer: (a) 8 cm (b) 20 cm^2

4 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-4x - 3| > -3x + 3$.

Critical values: $-4x - 3 = -3x + 3$ gives $x = -6$; $-4x - 3 = -(-3x + 3)$ gives $x = 0$
Sketch $y = |-4x - 3|$ and $y = -3x + 3$, and compare the graphs at $x = -6$ and $x = 0$.
$|-4x - 3| > -3x + 3$ when $x < -6$ or $x > 0$
Answer: $x < -6$ or $x > 0$

5 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The height of the tide in a harbour is modelled by $H = 11 + 4\sin(\frac{\pi t}{12})$, where t is in hours after midnight.

Find (a) the greatest and least values, (b) the values of t with $0 \leq t \leq 24$ when it equals 13 m.

(a) sin ranges from -1 to 1: greatest $11 + 4 = 15$, least $11 - 4 = 7$

(b) $11 + 4\sin(\frac{\pi t}{12}) = 13$, so $\sin(\frac{\pi t}{12}) = \frac{1}{2}$

$\frac{\pi t}{12} = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, so $t = 2$ or 10

Answer: (a) 15 m, 7 m (b) t = 2 and 10

6 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 50 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 20 to t = 40.
The floors are 630 m apart. Find V, and the deceleration in the final stage.

Area $= \frac{1}{2} \times (50 + 20) \times V = 630$
$70V = 1260$ so $V = 18$
Deceleration $= 18 \div 10$

Answer: V = 18; deceleration 1.8 m s^{-2}

7 Finding the mean or standard deviation (All boards)

The mass X g of a flour bag is normal with s.d. 5.
Given P(X > 8.4) = 0.01, find the mean (3 s.f.).

P(X > 8.4) = 0.01, so P(X < 8.4) = 0.99

$z = -2.326$: $\frac{8.4 - \mu}{5} = -2.326$

$\mu = 8.4 + 2.326 \times 5 = 20.0$

Answer: $\mu = 20.0$

8 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (6 kg) and B (9 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 4.5 m above the ground, then released.
Take g = 9.8 m s^{-2}.
Find the speed with which B hits the ground.

$a = (9 - 6)g \div (9 + 6) = 1.96$

$v^2 = 0 + 2 \times 1.96 \times 4.5 = 17.64$

$v = 4.2$

Answer: 4.2 m s^{-1}

9 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x - 5)x(x + 5) < 0$.

Critical values: $x = -5$, $x = 0$, $x = 5$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.
Reading from the sketch: $x < -5$ or $0 < x < 5$

Answer: $x < -5$ or $0 < x < 5$

10 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write 21 cos θ + 20 sin θ in the form R cos(θ − α), with R > 0 and 0 < α < 90°.
(b) Hence find its maximum value and the smallest θ ≥ 0 at which it occurs. Give α to 1 d.p.

$R = \sqrt{21^2 + 20^2} = 29$, $\tan\alpha = \frac{20}{21}$ so $\alpha = 43.6°$

Maximum is R when $\cos(\theta - \alpha) = 1$, i.e. $\theta = \alpha$

Answer: (a) 29 cos(θ − 43.6°) (b) max 29 at θ = 43.6°