Day 349 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $(\frac{32}{243})^{\frac{1}{5}}$.

$\frac{32}{243} = (\frac{2}{3})^5$, so the expression $= (\frac{2}{3})^1$

$= \frac{2}{3}$

Answer: $\frac{2}{3}$

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has area 115.2 cm^2 and angle 1.6 radians.
Find (a) the radius, (b) the perimeter of the sector.

$\frac{1}{2}r^2\theta = 115.2$, so $r^2 = \frac{2 \times 115.2}{1.6} = 144$, $r = 12$

Perimeter $= 2r + r\theta = 24 + 19.2 = 43.2$ cm

Answer: (a) 12 cm (b) 43.2 cm

3 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 50 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 15 to t = 35.
The floors are 525 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (50 + 20) × V = 525
70 V = 1050 so V = 15
Deceleration = 15 ÷ 15

Answer: V = 15; deceleration 1 m s^{-2}

4 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $-2x^2 - 12x - 27 < 0$ for all real values of x.

Complete the square: $-2x^2 - 12x - 27 = -2(x + 3)^2 - 9$
$(x + 3)^2 \geq 0$ for all real x, so $-2(x + 3)^2 \leq 0$.
Adding -9: the expression is ≤ -9, so < 0 for all real x.

Answer: $-2x^2 - 12x - 27 < 0$ for all x

5 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 2 to 18): $y = 60 - 2.5x$.
x is the temperature, y the gas used per day.
(a) Interpret the gradient.
(b) Estimate y when x = 20 and comment.

(a) The gradient: for each extra °C rise in temperature, y is predicted to decrease by 2.5
(b) $y = 60 + (-2.5) \times 20 = 10$
x = 20 lies outside the range of the data (extrapolation), so the estimate may be unreliable

Answer: (a) y decreases by 2.5 per extra °C rise in temperature (b) 10, unreliable (extrapolation)

6 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of $(1 + 4x)^9$.

Term in $x^2 = 36 \times 1^7 \times (4x)^2$
$= 36 \times 1 \times 16\ x^2$

Answer: 576

Day 349 **A Level Maths: B Tier 10-a-day: Model Solutions**

7 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf f(x) = kx for $0 \leq x \leq 6$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 1). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^2/2]$ from 0 to 6 = $k \times \frac{6^2}{2} = 1$, so $k = \frac{1}{18}$

(b) $P(X > 1) = \int_1^6 \frac{1}{18} x\,dx = 1 - \frac{1^2}{6^2} = \frac{35}{36}$

(c) $E(X) = \int_0^6 x f(x)\,dx = \frac{1}{18} \times \frac{6^3}{3} = 4$

Answer: (a) $k = \frac{1}{18}$ (b) $\frac{35}{36}$ (c) 4

8 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^1 (x + 2)e^{3x}\,dx$.

Let $u = x + 2$ and $\frac{dv}{dx} = e^{3x}$, so $\frac{du}{dx} = 1$ and $v = \frac{e^{3x}}{3}$

Integral = $[uv] - \int v\frac{du}{dx}\,dx$

$= \left[\frac{(x + 2)e^{3x}}{3}\right]_0^1 - \int_0^1 \frac{e^{3x}}{3}\,dx$

$= \frac{8e^3}{9} - \frac{5}{9}$

Answer: $\frac{8e^3}{9} - \frac{5}{9}$

9 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trunk of mass 10 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{3}{5}$.

The coefficient of friction is 0.5. Take g = 9.8 m s^{-2}. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 98 - \frac{4}{5}T$

Horizontal: $T\cos\alpha = \mu R$: $\frac{3}{5}T = 0.5(98 - \frac{4}{5}T)$

$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.5 \times 10 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)

Answer: 49 N

10 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $4\sin^2 x + 7\sin x + 3 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

$(4\sin x + 3)(\sin x + 1) = 0$, so $\sin x = -\frac{3}{4}$ or $\sin x = -1$

$\sin x = -\frac{3}{4}$: x = 228.6°, 311.4°

$\sin x = -1$: x = 270.0°

Answer: x = 228.6°, 270.0°, 311.4°