Day 348 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term a and common ratio r, where $|r| < 1$.

The sum to infinity is $\frac{36}{5}$ times the second term.

Find the possible values of r.

$\frac{a}{1-r} = \frac{36}{5} \times ar$, so $1 = \frac{36}{5} r(1-r)$

$\frac{36}{5}r^2 - \frac{36}{5}r + 1 = 0$

Multiply by 5: $36r^2 - 36r + 5 = 0$

$r = \frac{1}{6}$ or $r = \frac{5}{6}$ (both satisfy $|r| < 1$)

Answer: $r = \frac{1}{6}$ or $r = \frac{5}{6}$

2 Particle on a table joined to a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg lies on a smooth horizontal table. It is attached by a light string over a smooth pulley at the edge of the table to a particle of mass 8 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released from rest. Find the acceleration and the tension in the string.

Hanging: $8g - T = 8a$

Table: $T = 6a$

$8g = 14a$, so $a = 5.6$

$T = 6 \times 5.6$

Answer: $a = 5.6$ m s^{-2}; $T = 33.6$ N

3 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ and B has position vector $\begin{pmatrix} -8 \\ -10 \\ 7 \end{pmatrix}$.

The point P lies on AB with AP : PB = 2 : 1.

Find the position vector of P.

$\overrightarrow{AB} = \begin{pmatrix} -9 \\ -9 \\ 3 \end{pmatrix}$

$\overrightarrow{AP} = \frac{2}{3}\overrightarrow{AB} = \begin{pmatrix} -6 \\ -6 \\ 2 \end{pmatrix}$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \begin{pmatrix} -5 \\ -7 \\ 6 \end{pmatrix}$

Answer: $\begin{pmatrix} -5 \\ -7 \\ 6 \end{pmatrix}$

4 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the marks in a mock exam for two groups.

(a) Find the interquartile range of each group.

(b) Compare the two distributions.

(a) Boys: $50 - 25 = 25$. Girls: $30 - 20 = 10$

(b) Medians: 40 (Boys) and 25 (Girls), so Boys has the higher median

IQRs: Boys has the larger IQR, so its scores are more spread out

Answer: (a) 25 and 10 (b) Boys has the higher median; Boys is more spread out (larger IQR), Girls more consistent

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5 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 36 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{12}{13}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 36$, so $P \times \frac{12}{13} = 36$

$P = 39$

Resolve vertically: $Q = P\sin\theta = 39 \times \frac{5}{13}$

Answer: P = 39 N; Q = 15 N

6 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 1 to 9): $y = 12 + 25x$.
x is the hours of sunshine, y the number of ice creams sold.
(a) Interpret the gradient.
(b) Estimate y when x = 12 and comment.

(a) The gradient: for each extra hour of sunshine, y is predicted to increase by 25
(b) $y = 12 + (25) \times 12 = 312$
x = 12 lies outside the range of the data (extrapolation), so the estimate may be unreliable
Answer: (a) y increases by 25 per extra hour of sunshine (b) 312, unreliable (extrapolation)

7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of
$(1 - 2x)(2 + 2x)^4$.

$(2 + 2x)^4$ has x^2 coefficient 96 and x^3 coefficient 64
x^3 term of the product $= 1 \times 64 + (-2) \times 96$
$= -128$
Answer: -128

8 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix}0\\-1\\3\end{pmatrix}$,

$\begin{pmatrix}1\\-1\\4\end{pmatrix}$ and $\begin{pmatrix}2\\-1\\r\end{pmatrix}$ respectively.

Given that A, B, C are collinear, find r.

$\overrightarrow{AB} = \begin{pmatrix}1\\0\\1\end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix}2\\0\\r-3\end{pmatrix}$

$\overrightarrow{AC} = \lambda\overrightarrow{AB}$: component 1 gives $2 = \lambda \times 1$, so $\lambda = 2$
$r - 3 = 2 \times 1$, so $r = 5$
Answer: r = 5

9 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x + 3$ meets the circle $(x + 3)^2 + (y - 1)^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -x + 3$: $(x + 3)^2 + (-x + 2)^2 = 25$
$x^2 + x - 6 = 0$
$(x - 2)(x + 3) = 0$, so $x = -3$ or $x = 2$
$y = -x + 3$: A = (−3, 6), B = (2, 1)
$AB^2 = 5^2 + (-5)^2 = 50$
Answer: A(−3, 6), B(2, 1), AB = $5\sqrt{2}$

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10 Simplify using double angle formulae (All boards)

Simplify $\frac{\cos 6x}{\cos 3x + \sin 3x}$,

giving your answer in its simplest form.

$\cos 6x = \cos^2 3x - \sin^2 3x = (\cos 3x - \sin 3x)(\cos 3x + \sin 3x)$

so the expression is $\cos 3x - \sin 3x$

Answer: $\cos 3x - \sin 3x$