Day 347 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Basic trig equations (degrees) (All boards)

Solve $\sin x = -\frac{\sqrt{2}}{2}$ for $0 \leq x \leq 360°$.

Principal value $x = -45°$
Second solution: $180° - -45° = 225°$
Answer: x = 225°, 315°

2 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) $y = f(-x)$
(b) $y = 3f(x)$
(c) $y = f(\frac{x}{4})$

(a) $y = f(-x)$ is a reflection in the y-axis.
(b) $y = 3f(x)$ is a stretch, scale factor 3, parallel to the y-axis.
(c) $y = f(\frac{x}{4})$ is a stretch, scale factor 4, parallel to the x-axis.

Answer: (a) reflection in the y-axis (b) stretch, scale factor 3, parallel to the y-axis (c) stretch, scale factor 4, parallel to the x

3 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 5 kg rests on a rough horizontal floor, with coefficient of friction 0.2. A horizontal force of 12 N is applied to the box.
Take $g = 9.8$ m s^{-2}.
Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 5 \times 9.8 = 49$, maximum friction $\mu R = 0.2 \times 49 = 9.8$
$12 > 9.8$ so the box moves, $F = 9.8$
$12 - 9.8 = 5a$
Answer: It moves; a = 0.44 m s^{-2}

4 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

8 values of x are coded using $y = \frac{(x - 20)}{10}$.

$\Sigma y = 29$ and $\Sigma y^2 = 285$.
Find the mean and standard deviation of x (3 s.f.).

Mean of $y = 29 \div 8 = 3.625$
Mean of $x = 20 + 10 \times 3.625 = 56.2$
Variance of $y = \frac{285}{8} - (3.625)^2 = 22.48$
SD of $y = 4.742$, so SD of $x = 10 \times 4.742 = 47.4$
Answer: mean 56.2, standard deviation 47.4

5 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 3)^2 + (y + 4)^2 = 100$.
The point P(−5, −10) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (3, −4). Gradient of radius CP = $\frac{-6}{-8} = \frac{3}{4}$

The tangent is perpendicular to the radius: gradient = $-\frac{4}{3}$

$y + 10 = -\frac{4}{3}(x + 5)$

Multiply out and collect terms: $4x + 3y + 50 = 0$
Answer: 4x + 3y + 50 = 0

Day 347 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 20 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.

A particle of mass 1 kg is attached 2 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(3) = 20\,g(1.5) + 1\,g(1.5)$

$T_2 = 102.9$

$T_1 + T_2 = 21\,g$ so $T_1 = 205.8 - 102.9$

Answer: Near A: 102.9 N; near B: 102.9 N

7 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations

$x = 2t^2$, $y = 4t$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the tangent at $t = -2$.

$\frac{dx}{dt} = 4t$, $\frac{dy}{dt} = 4$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{t}$; at $t = -2$ it is $-\frac{1}{2}$

Point: $(8, -8)$

$y + 8 = -\frac{1}{2}(x - 8)$, so $y = -\frac{1}{2}x - 4$

Answer: $\frac{dy}{dx} = \frac{1}{t}$; tangent $y = -\frac{1}{2}x - 4$

8 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.

P(X < 74) = 0.0668 and P(X > 84) = 0.1587.

Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$

$74 = \mu + (-1.500)\sigma$ and $84 = \mu + 0.9998\sigma$

Subtract: $10 = 2.500\,\sigma$, so $\sigma = 4.00$

$\mu = 74 - (-1.500)(4.000) = 80.0$

Answer: $\mu = 80.0$, $\sigma = 4.00$

9 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, with $R > 0$ and $0 < \alpha < 90°$.

Hence state the minimum value of the expression. Give α to 1 d.p.

$R\cos\alpha = 1$, $R\sin\alpha = 1$

$R = \sqrt{2}$ and $\tan\alpha = \frac{1}{1}$, so $\alpha = 45.0°$

Minimum value of $R\sin(\theta - \alpha)$ is $-R$

Answer: $\sqrt{2}\sin(\theta - 45.0°)$; minimum $-\sqrt{2}$

10 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 4 and common ratio 2.

Find the smallest value of n for which the sum of the first n terms exceeds 20000.

$S_n = 4(2^n - 1) > 20000$

$2^n > 5001$

$n > \frac{\lg 5001}{\lg 2} = 12.288$

So the smallest integer n is 13

Answer: n = 13