Day 346 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 8 kg rests on a rough horizontal floor, with coefficient of friction 0.5. A horizontal force of 31 N is applied to the box.

Take $g = 9.8$ m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 8 \times 9.8 = 78.4$, maximum friction $\mu R = 0.5 \times 78.4 = 39.2$

$31 < 39.2$ so the box stays at rest

Friction balances the applied force: $F = 31$

Answer: Does not move; friction = 31 N

2 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a} \cdot \mathbf{b} = (3)(-4) + (0)(-2) + (-2)(3) = -18$

$|\mathbf{a}| = \sqrt{13}$, $|\mathbf{b}| = \sqrt{29}$

$\cos\theta = \dfrac{-18}{\sqrt{13}\sqrt{29}} = -0.9270$

$\theta = 158.0°$

Answer: 158.0°

3 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + 7n + 14$ is even for all integers n.

$n^2 + 7n = n(n + 7)$

As 7 is odd, n and n + 7 have opposite parity,

so one of them is even and the product $n(n + 7)$ is even.

Even + 14 = even, so $n^2 + 7n + 14$ is even.

Answer: n(n + 7) is even, so the expression is even

4 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 11 pairs of data, $S_{xx} = 40$, $S_{yy} = 80$ and $S_{xy} = -48$.

Find the PMCC (3 s.f.) and describe the correlation.

$r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \dfrac{-48}{\sqrt{40 \times 80}}$

$= \dfrac{-48}{56.569} = -0.849$

r is negative, so there is strong negative correlation

Answer: r = −0.849, strong negative correlation

5 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $(7\mathbf{i} + 6\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-2}.

Find $\mathbf{P}$, and its magnitude to 3 significant figures.

$\mathbf{F} = m\mathbf{a}$: $(7\mathbf{i} + 6\mathbf{j}) + \mathbf{P} = 3(-2\mathbf{i} + 2\mathbf{j}) = (-6\mathbf{i} + 6\mathbf{j})$

$\mathbf{P} = -13\mathbf{i}$

$|\mathbf{P}| = \sqrt{(-13^2 + 0^2)}$

Answer: P = (−13i) N; |P| = 13.0 N

6 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = 4x - \ln x$, and determine its nature.

$\frac{dy}{dx} = \frac{4x - 1}{x} = 0$

$x = \frac{1}{4}$

$\frac{d^2y}{dx^2} = \frac{1}{x^2}$

At $x = \frac{1}{4}$: $\frac{d^2y}{dx^2} = 16 > 0$, so a minimum

Answer: $(\frac{1}{4}, 1 + \ln 4)$ minimum

7 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$. A and B are the turning points. Find the turning points of:
(a) $y = 2f(x)$ (b) $y = f(-x)$

(a) $y = 2f(x)$: A$(-3, 1)$ maps to $(-3, 2)$, B$(1, -4)$ maps to $(1, -8)$
(b) $y = f(-x)$: A$(-3, 1)$ maps to $(3, 1)$, B$(1, -4)$ maps to $(-1, -4)$
Answer: (a) $(-3, 2)$, $(1, -8)$ (b) $(3, 1)$, $(-1, -4)$

8 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = \frac{3}{4}$ and θ is acute,

find the exact value of $\sec\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\tan\theta = \frac{3}{4}$, so $\sec^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\sec\theta = \frac{5}{4}$ (positive as θ is acute)

Answer: $\frac{5}{4}$

9 Normal approximation to the binomial (AQA, OCR, CIE)

$X \sim B(200, 0.2)$. Use a normal approximation to find $P(40 \leq X \leq 44)$, to 3 significant figures.

$Y \sim N(np, np(1-p)) = N(40.00, 32.00)$, so $\sigma = \sqrt{32.00} = 5.657$
Use a continuity correction.
$P(40 \leq X \leq 44) \approx P(39.5 < Y < 44.5) = 0.322$
Answer: 0.322

10 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1 - u_n}$, $u_1 = 4$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{100} and $\sum_{r=1}^{100} u_r$.

(a) $u_2 = -\frac{1}{3}$, $u_3 = \frac{3}{4}$, $u_4 = 4$
$u_4 = u_1$ so the sequence repeats with period 3
(b) $100 = 3 \times 33 + 1$, so $u_{100} = u_1 = 4$
Sum of one cycle $= \frac{53}{12}$, so the sum $= 33 \times \frac{53}{12} + 4 = \frac{599}{4}$

Answer: period 3; $u_{100} = 4$; sum $= \frac{599}{4}$