Day 345 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 5 < -5x + 2$.

$6x < 7$

Divide by 6: $x < \frac{7}{6}$

Answer: $x < \frac{7}{6}$

2 Poisson distribution (CIE)

X ~ Po(0.8), modelling the number of emails received per hour.
Find, to 3 significant figures,
(a) $P(X \geq 3)$
(b) $P(X \leq 3)$

(a) $P(X \geq 3) = 1 - P(X \leq 2) = 0.0474$
(b) $P(X \leq 3) = P(X = 0) + ... + P(X = 3) = 0.991$
Answer: (a) 0.0474 (b) 0.991

3 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 2x - 3$. Use the Newton-Raphson method with
$x_0 = 1$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 3x^2 + 2$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1 - \frac{0}{5.000} = 1.0000$

$x_2 = 1.0000 - \frac{0}{5.000} = 1.0000$

Answer: $x_1 = 1.0000$, $x_2 = 1.0000$

4 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (2, 0), (0, 1) and (−1, −1).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB $= \frac{-1}{2} = -\frac{1}{2}$

Gradient BC $= \frac{-2}{-1} = 2$

$(-\frac{1}{2}) \times 2 = -1$, so AB and BC are perpendicular.

(b) AB $= \sqrt{5}$ and BC $= \sqrt{5}$
Area $= \frac{1}{2} \times AB \times BC = \frac{5}{2}$

Answer: (b) area $= \frac{5}{2}$

5 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{40}{41}$ and $\sin B = \frac{7}{25}$, where A and B are acute,
find the exact value of $\cos(A - B)$.

$\cos A = \frac{9}{41}$ and $\cos B = \frac{24}{25}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$= \frac{9}{41} \times \frac{24}{25} + \frac{40}{41} \times \frac{7}{25} = \frac{496}{1025}$

Answer: $\frac{496}{1025}$

6 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is kicked from horizontal ground with speed 24.5 m s^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{3}{4}$.

Take g = 9.8 m s^{-2}.

Find (a) the time of flight, (b) the horizontal range.

cos θ = 4/5, sin θ = 3/5: horizontal 19.6, vertical 14.7

(a) $0 = 14.7\,t - 4.9t^2$, $t = 14.7 \div 4.9$

(b) Range = 19.6 × 3

Answer: (a) 3 s (b) 58.8 m

7 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write 12 cos θ + 5 sin θ in the form R cos($\theta - \alpha$), with R > 0 and $0 < \alpha < 90°$.

(b) Hence find its maximum value and the smallest $\theta \geq 0$ at which it occurs. Give α to 1 d.p.

$R = \sqrt{12^2 + 5^2} = 13$, $\tan\alpha = \frac{5}{12}$ so $\alpha = 22.6°$

Maximum is R when $\cos(\theta - \alpha) = 1$, i.e. $\theta = \alpha$

Answer: (a) 13 cos(θ − 22.6°) (b) max 13 at θ = 22.6°

8 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ax^n$. A plot of $\log_{10} y$ against $\log_{10} x$ is a straight line with intercept 1.00 and gradient 0.5.

Find a (2 s.f.) and n, then estimate y when x = 4 (3 s.f.).

$\log_{10} y = \log_{10} a + n \log_{10} x$

n = gradient = 0.5; $\log_{10} a = 1.00$, so $a = 10^{1.00} = 10$

$y = 10.00 \times 4^{0.5} = 20.0$

Answer: a = 10, n = 0.5, y = 20.0

9 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1 - u_n}$, $u_1 = 3$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{25} and $\sum_{r=1}^{25} u_r$.

(a) $u_2 = -\frac{1}{2}$, $u_3 = \frac{2}{3}$, $u_4 = 3$

$u_4 = u_1$ so the sequence repeats with period 3

(b) $25 = 3 \times 8 + 1$, so $u_{25} = u_1 = 3$

Sum of one cycle = $\frac{19}{6}$, so the sum = $8 \times \frac{19}{6} + 3 = \frac{85}{3}$

Answer: period 3; u_{25} = 3; sum = $\frac{85}{3}$

10 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 10 kg is held on a rough plane ($\tan\alpha = \frac{4}{3}$, $\mu = 0.25$) by a force P N up the plane, as shown.

Take g = 9.8 m s^{-2}. The block is about to move down the plane. Find P.

R = mg cos α = 98 × 3/5 = 58.8

Friction (limiting) F = μR = 14.7

Moving down, friction acts up: P + F = mg sin α

mg sin α = 78.4

Answer: 63.7 N