Day 344 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Normal distribution probabilities (All boards)

The mass X g of a flour bag is normal with mean 75, s.d. 4.
Find, to 3 s.f., the probability a value is
(a) less than 83,
(b) between 83 and 85.

$X \sim N(75, 4^2)$
(a) $P(X < 83) = 0.977$
(b) $P(83 < X < 85) = 0.9938 - 0.9772 = 0.0165$
Answer: (a) 0.977 (b) 0.0165

2 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 2 kg is pulled along rough horizontal ground by a horizontal force of 11.84 N. It accelerates at 2 m s^{-2}. Take g = 9.8 m s^{-2}.
Find the coefficient of friction between the sledge and the ground.

$R = mg = 2 \times 9.8 = 19.6$
Resultant: $11.84 - F = 2 \times 2$, so $F = 7.84$
$F = \mu R$: $\mu = 7.84 \div 19.6$
Answer: 0.4

3 Independent and mutually exclusive events (All boards)

A and B are independent events with $P(A) = 0.3$ and $P(A \cup B) = 0.58$.
(a) Find P(B).
(b) Find $P(A' \cap B')$.

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$
$0.58 = 0.3 + P(B)(1 - 0.3)$, so $P(B) = 0.28 \div 0.7 = 0.4$
(b) $P(A' \cap B') = 1 - P(A \cup B) = 0.42$
Answer: (a) 0.4 (b) 0.42

4 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(4 + 2x)^5$ in ascending powers of x, up to and including the term in x^3.

$(4 + 2x)^5 = 4^5 + 5 \times 4^4(2x) + 10 \times 4^3(2x)^2 + 10 \times 4^2(2x)^3$
$= 1024 + 2560x + 2560x^2 + 1280x^3 + ...$
Answer: $1024 + 2560x + 2560x^2 + 1280x^3 + ...$

5 Particle in equilibrium: string and horizontal force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 3 kg hangs in equilibrium on a light string inclined at α to the vertical, held by a horizontal force P N, as shown. $\tan \alpha = \frac{12}{5}$.

Take g = 9.8 m s^{-2}.
Find the tension in the string and the value of P.

$\cos \alpha = 5/13$, $\sin \alpha = 12/13$
Vertical: $T \cos \alpha = mg = 29.4$, so $T = 29.4 \times 13/5$
Horizontal: $P = T \sin \alpha = 76.44 \times 12/13$
Answer: T = 76.44 N; P = 70.56 N

Day 344 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = t^2$, $y = t^3$.
Find $\frac{dy}{dx}$ in terms of t, and the equation of the tangent at t = 1.

$\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 3t^2$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t}{2}$; at t = 1 it is $\frac{3}{2}$

Point: (1, 1)

$y - 1 = \frac{3}{2}(x - 1)$, so $y = \frac{3}{2}x - \frac{1}{2}$

Answer: $\frac{dy}{dx} = \frac{3t}{2}$; tangent $y = \frac{3}{2}x - \frac{1}{2}$

7 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 4 and common ratio 3.
Find the smallest value of n for which the sum of the first n terms exceeds 10000.

$S_n = \frac{4(3^n - 1)}{2} > 10000$

$3^n > 5001$

$n > \frac{\lg 5001}{\lg 3} = 7.753$

So the smallest integer n is 8

Answer: n = 8

8 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), find an approximation of the
form $1 + p\theta + q\theta^2$ for
$\sin 2\theta + \cos 4\theta + 3\tan\theta$.

$\sin 2\theta \approx 2\theta$, $\cos 4\theta \approx 1 - \frac{16\theta^2}{2}$, $\tan\theta \approx \theta$

Sum $\approx 1 + 2\theta - \frac{16\theta^2}{2} + 3\theta$

$\approx 1 + 5\theta - 8\theta^2$

Answer: p = 5, q = −8

9 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is (−6, 4) and B is (−2, 0).
Find an equation of the circle in the form
$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = (−4, 2)

$\text{Radius}^2 = (-6 - (-4))^2 + (4 - 2)^2 = 8$

Answer: $(x + 4)^2 + (y - 2)^2 = 8$

10 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 14 cm and arc length 7 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

$s = r\theta$, so $\theta = \frac{7}{14} = 0.5$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 14^2 \times 0.5 = 49$

Answer: (a) 0.5 rad (b) 49 cm^2