Day 343 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moves in a straight line with constant acceleration. Its final speed is 47 m s^{-1}, its acceleration is 5 m s^{-2} and the motion lasts 9 s.
Find the initial speed.

$v = u + at$
$47 = u + 5 \times 9$
$u = 47 - 45$
Answer: 2 m s^{-1}

2 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ q \\ -7 \end{pmatrix}$ respectively.
Given that A, B, C are collinear, find q.

$\text{AB} = \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$, $\text{AC} = \begin{pmatrix} -3 \\ q-2 \\ -6 \end{pmatrix}$
AC = λAB: component 1 gives $-3 = \lambda \times -1$, so $\lambda = 3$
$q - 2 = 3 \times -1$, so $q = -1$
Answer: q = −1

3 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should y = 5 + x not be used for x = 14?

(a) The points lie close to a straight line rising: strong positive correlation
(b) The data cover x = 1 to 10; x = 14 is outside this range
so using the line would be extrapolation and may be unreliable
Answer: (a) strong positive correlation (b) extrapolation outside the range of the data

4 Double angle equations (All boards)

Solve $\cos 2x + 5 \cos x + 3 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $2\cos^2 x - 1 + 5 \cos x + 3 = 0$
$(2 \cos x + 1)(\cos x + 2) = 0$
$\cos x = -\frac{1}{2}$: x = 120.0°, 240.0°
$\cos x = -2$ has no solutions
Answer: x = 120.0°, 240.0°

5 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(−3, 1) and B(0, 2), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $\text{CA}^2 = \text{CB}^2$:
$9 + (h - 1)^2 = 0 + (h - 2)^2$
Expanding and cancelling h^2: $-2h + 10 = -4h + 4$
$h = -3$
$r^2 = \text{CA}^2 = 25$
Answer: centre (0, −3), $x^2 + (y + 3)^2 = 25$

6 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 4 s, (b) the distance between the stations.

(a) a = gradient = 12 ÷ 4
(b) Distance = area of trapezium = ½ × (27 + 15) × 12
Answer: (a) 3 m s^{-2} (b) 252 m

7 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), find an approximation of the
form $1 + p\theta + q\theta^2$ for
$\sin 4\theta + \cos 2\theta + 2\tan\theta$.

$\sin 4\theta \approx 4\theta$, $\cos 2\theta \approx 1 - \frac{4\theta^2}{2}$, $\tan\theta \approx \theta$

$\text{Sum} \approx 1 + 4\theta - \frac{4\theta^2}{2} + 2\theta$

$\approx 1 + 6\theta - 2\theta^2$
Answer: p = 6, q = −2

8 Binomial critical region (All boards)

X ~ B(25, p). H_0: p = 0.25 is to be tested against H_1: p > 0.25 at the 5% significance level.
Find the critical region and the actual significance level.

Under H_0, X ~ B(25, 0.25)
P(X ≥ 11) = 0.0297 ≤ 0.05
P(X ≥ 10) = 0.0713 > 0.05
The critical region is X ≥ 11
Answer: X ≥ 11; actual significance level 2.97%

9 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 4x − 3y = k is a tangent to the circle
$(x + 2)^2 + (y - 1)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (4, −3).
A unit step in this direction is (4, −3)/5, so the points of contact are
(−2, 1) ± 10 × (4, −3)/5 = (−2, 1) ± (8, −6)
T = (6, −5) or (−10, 7), and k = 4x − 3y gives:
k = 39 or k = −61
Answer: k = 39 or k = −61

10 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-6x^2 + 37x - 54}{(x - 2)(x - 4)^2}$ in partial fractions.

$$\frac{-6x^2 + 37x - 54}{(x - 2)(x - 4)^2} \equiv \frac{A}{(x - 2)} + \frac{B}{(x - 4)} + \frac{C}{(x - 4)^2}$$

$-6x^2 + 37x - 54 \equiv A(x - 4)^2 + B(x - 2)(x - 4) + C(x - 2)$
x = 2 gives A = −1; x = 4 gives C = −1
Coefficients of x^2: −6 = A + B, so B = −5
Answer: $-\frac{1}{(x - 2)} - \frac{5}{(x - 4)} - \frac{1}{(x - 4)^2}$