Day 341 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-10 < 4x < 4$.

Subtract 0 from each part: $-10 < 4x < 4$

Divide each part by 4: $-\frac{5}{2} < x < 1$

Answer: $-\frac{5}{2} < x < 1$

2 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected vertically upwards from ground level with speed 39.2 m s^{-1}.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height reached, (b) the total time before it returns to the ground.

(a) At the top v = 0: $0 = 39.2^2 - 2 \times 9.8 \times H$
$H = 1536.64 \div 19.6 = 78.4$
(b) s = 0: $0 = 39.2\,t - 4.9t^2$, so $t = 39.2 \div 4.9$
Answer: (a) 78.4 m (b) 8 s

3 Poisson distribution (CIE)

X ~ Po(1.2), modelling the number of emails received per hour.
Find, to 3 significant figures,
(a) $P(X \leq 2)$
(b) $P(X \geq 2)$

(a) $P(X \leq 2) = P(X = 0) + ... + P(X = 2) = 0.879$
(b) $P(X \geq 2) = 1 - P(X \leq 1) = 0.337$
Answer: (a) 0.879 (b) 0.337

4 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 5 \cot 2x$ with respect to x,
and find the exact gradient when $x = \frac{\pi}{8}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = -10 \operatorname{cosec}^2 2x$
At $x = \frac{\pi}{8}$: $\frac{dy}{dx} = -20$
Answer: $\frac{dy}{dx} = -10 \operatorname{cosec}^2 2x$; gradient -20

5 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 4 and common ratio 4.
Find the smallest value of n for which the sum of the first n terms exceeds 50000.

$S_n = \frac{4(4^n - 1)}{3} > 50000$
$4^n > 37501$
$n > \frac{\lg 37501}{\lg 4} = 7.597$
So the smallest integer n is 8
Answer: n = 8

Day 341 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Rates of change (connected rates) (All boards)

The volume of a sphere increases at a constant rate of 100 $cm^3 s^{-1}$.
Find the rate at which the radius is increasing when r = 10 cm, to 3 s.f.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = \frac{100}{4\pi r^2}$

When r = 10: $\frac{dr}{dt} = \frac{1}{4\pi} = 0.07958$

Answer: 0.0796 cm/s

7 Binomial mean and variance (All boards)

X has a binomial distribution with mean 7.5 and variance 1.875.
(a) Find n and p.
(b) Find P(X = 7) to 3 significant figures.

(a) np = 7.5 and np(1 − p) = 1.875
1 − p = 1.875 ÷ 7.5 = 0.25, so p = 0.75 and n = 7.5 ÷ 0.75 = 10
(b) $P(X = 7) = {}^{10}C_7 \times 0.75^7 \times 0.25^3 = 0.250$

Answer: (a) n = 10, p = 0.75 (b) 0.250

8 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 11 cm and angle 60°.
Find the perimeter of the sector, in terms of π.

$\theta = \frac{\pi}{3}$ radians

Arc length = $r\theta = \frac{11\pi}{3}$ cm

Perimeter = 2r + arc = $\frac{11\pi}{3}$ + 22 cm

Answer: $\frac{11\pi}{3}$ + 22 cm

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{3}$ is irrational, prove by contradiction that $1 + 5\sqrt{3}$ is irrational.

Assume $1 + 5\sqrt{3} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $5\sqrt{3} = \frac{r}{s} - 1 = \frac{r - 1s}{s}$, so $\sqrt{3} = \frac{r - 1s}{5s}$.

The right-hand side is a ratio of integers, so it is rational.
This contradicts $\sqrt{3}$ being irrational, so $1 + 5\sqrt{3}$ is irrational.

Answer: $1 + 5\sqrt{3}$ is irrational

10 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected from level ground with horizontal velocity component 25 m s^{-1} and vertical velocity component 24.5 m s^{-1} upwards.
Take g = 9.8 m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 24.5\,t - 4.9t^2$, so t = 24.5 ÷ 4.9
(b) Range = horizontal speed × time = 25 × 5

Answer: (a) 5 s (b) 125 m