Day 340 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "8n + 11 is prime for every positive integer n."
Show that this statement is false by finding a counter-example.

Try n = 1, 2, 3, ... until the value is not prime.
n = 2 gives 8 × 2 + 11 = 27
27 = 3 × 3 × 3, which is not prime.
Answer: n = 2 (value 27 is not prime)

2 Binomial probabilities (All boards)

X ~ B(21, 0.1). Find, to 3 significant figures,
(a) P(X = 1)
(b) P(X > 1)

(a) $P(X = 1) = {}^{21}C_1 \times 0.1^1 \times 0.9^{20} = 0.255$
(b) $P(X > 1) = 1 - P(X \leq 1)$
(b) = 0.635
Answer: (a) 0.255 (b) 0.635

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 7 cm and area $\frac{147\pi}{4}$ cm^2.

Find the angle of the sector in degrees.

$\frac{1}{2}r^2\theta = \frac{147\pi}{4}$, so $\theta = \frac{2 \times \frac{147\pi}{4}}{7^2} = \frac{3\pi}{2}$ radians

$\theta = \frac{3\pi}{2} \times \frac{180}{\pi} = 270°$
Answer: 270°

4 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 3t^2 - 3t + 4$ m s^{-1}.
Find the distance moved between t = 2 and t = 6. (The particle does not change direction.)

$s = \int_2^6 (3t^2 - 3t + 4)\, dt$

$= [\, t^3 - \frac{3}{2}t^2 + 4t \,]$ from t = 2 to t = 6
= 186 − 10
Answer: 176 m

5 Rates of change (connected rates) (All boards)

The volume of a sphere increases at a constant rate of 20 cm^{3}s^{-1}.
Find the rate at which the radius is increasing when r = 3 cm, to 3 s.f.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = \frac{20}{4\pi r^2}$

When r = 3: $\frac{dr}{dt} = \frac{5}{9\pi} = 0.1768$
Answer: 0.177 cm/s

6 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(3 - 4x)(1 + 2x)^7$.

$(1 + 2x)^7$ has x^1 coefficient 14 and x^2 coefficient 84
x^2 term of the product = 3 × 84 + (−4) × 14
= 196
Answer: 196

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7 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $6t^2 - 36t + 48$ m s^{-1}.
Find the first time P is at instantaneous rest, and its distance from O then.

$v = 0$: $6(t - 2)(t - 4) = 0$, so $t = 2$ or $t = 4$
First time $t = 2$

$s = \int_0^2 v\,dt = [\,2t^3 - 18t^2 + 48t\,]$ from 0 to 2

Answer: t = 2 s; 40 m

8 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $3^{2x} + 3^x - 12 = 0$.

Let $y = 3^x$. Then $3^{2x} = y^2$, so $y^2 + y - 12 = 0$
$(y - 3)(y + 4) = 0$
$y = 3$ or $y = -4$
$3^x > 0$ for all x, so $y = -4$ is rejected. $3^x = 3$ gives $x = 1$.
Answer: x = 1

9 Binomial: smallest n (All boards)

Given that each component fails independently with probability 0.3, find the least number of components needed for the probability of at least one success to exceed 0.95.

P(at least one) $= 1 - 0.7^n > 0.95$
$0.7^n < 0.05$
$n > \frac{\log 0.05}{\log 0.7} = 8.399$
Answer: n = 9

10 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = x^2 + 2$, the x-axis
and the lines $x = 0$ and $x = 1$.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^1 y^2\,dx = \pi \int_0^1 (x^4 + 4x^2 + 4)\,dx$

$= \pi[\frac{x^5}{5} + \frac{4x^3}{3} + 4x]_0^1$

$= \frac{83\pi}{15}$

Answer: $\frac{83\pi}{15}$