**Day 34** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Basic trig equations (degrees) (All boards)

Solve $\cos x = \frac{\sqrt{3}}{2}$ for $0 \leq x \leq 360°$.

Principal value $x = 30°$
Second solution: $360° - 30° = 330°$
**Answer: x = 30°, 330°**

## 2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $3x^2 + 6x + 15$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the minimum point of $y = 3x^2 + 6x + 15$.

Take out the factor 3: $3x^2 + 6x + 15 = 3(x^2 + 2x) + 15$
$= 3[(x + 1)^2 - 1] + 15$
$= 3(x + 1)^2 + 12$
$(x + 1)^2 \geq 0$ with equality at $x = -1$, so the minimum is 12 at $x = -1$.
**Answer: (a) $3(x + 1)^2 + 12$ (b) $(-1, 12)$**

## 3 Solving exponential equations using logarithms (All boards)

Solve $3^x = 42$, giving your answer to 3 s.f.

Take logs: $x \ln 3 = \ln 42$
$x = \frac{\ln 42}{\ln 3} = 3.402$
**Answer: x = 3.40**

## 4 Binomial mean and variance (All boards)

X has a binomial distribution with mean 18.75 and variance 4.6875.
(a) Find n and p.
(b) Find P(X = 18) to 3 significant figures.

(a) $np = 18.75$ and $np(1 - p) = 4.6875$
$1 - p = 4.6875 \div 18.75 = 0.25$, so $p = 0.75$ and $n = 18.75 \div 0.75 = 25$
(b) $P(X = 18) = {}^{25}C_{18} \times 0.75^{18} \times 0.25^{7} = 0.165$
**Answer: (a) n = 25, p = 0.75 (b) 0.165**

## 5 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 4 kg is pulled along rough horizontal ground by a horizontal force of 15.76 N. It accelerates at 1 m $s^{-2}$.
Take $g = 9.8$ m $s^{-2}$.
Find the coefficient of friction between the box and the ground.

$R = mg = 4 \times 9.8 = 39.2$
Resultant: $15.76 - F = 4 \times 1$, so $F = 11.76$
$F = \mu R$: $\mu = 11.76 \div 39.2$
**Answer: 0.3**

## 6 Laws of logarithms: solving equations (All boards)

Solve $\log_5(x + 21) - \log_5(x - 3) = 2$.

$\log_5(\frac{x + 21}{x - 3}) = 2$, so $\frac{x + 21}{x - 3} = 5^2 = 25$
$x + 21 = 25(x - 3) = 25x - 75$
$24x = 96$, so $x = 4$
**Answer: x = 4**

## 7 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 1)^2 + (y - 2)^2 = 100$.
The point P(9, −4) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (1, 2). Gradient of radius CP = $\frac{-6}{8} = -\frac{3}{4}$

The tangent is perpendicular to the radius: gradient = $\frac{4}{3}$

$y + 4 = \frac{4}{3}(x - 9)$

Multiply out and collect terms: $4x - 3y - 48 = 0$

**Answer: $4x - 3y - 48 = 0$**

## 8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 5 g. Claimed mean 250 g; 50 packets have mean 250.9 g.
Test at the 5% level whether the true mean is greater than 250.

$H_0$: $\mu = 250$, $H_1$: $\mu > 250$. Under $H_0$, $\overline{X} \sim N(250, \frac{5^2}{50})$

$z = \frac{250.9 - 250}{5/\sqrt{50}} = 1.273$

$P(Z > 1.273) = 0.1015 > 0.05$

Do not reject $H_0$: there is insufficient evidence that the mean has increased

**Answer: do not reject $H_0$: insufficient evidence of an increase in the mean**

## 9 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 2t - t^2$ m s$^{-1}$, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 2 - 2t$

Maximum speed when $a = 0$: $t = 2 \div 2 = 1$

$v = 2 \times 1 - 1 \times 1^2$

**Answer: 1 m s$^{-1}$ at t = 1 s**

## 10 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre (−5, 3) and passes through the point (6, 6).
Find an equation of the circle.

$r^2 = (6 - (-5))^2 + (6 - 3)^2$

$= 121 + 9 = 130$

**Answer: $(x + 5)^2 + (y - 3)^2 = 130$**