Day 339 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A model boat moves so that its position vector relative to O at time t s is $\mathbf{r} = (t^2 + 3t)\mathbf{i} + (2t^3)\mathbf{j}$ metres.
Find the velocity when t = 2, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (2t + 3)\mathbf{i} + (6t^2)\mathbf{j}$
t = 2: $\mathbf{v} = (7)\mathbf{i} + (24)\mathbf{j}$
Speed = $\sqrt{(7^2 + 24^2)}$
Answer: $\mathbf{v} = (7)\mathbf{i} + (24)\mathbf{j}$ m s^{-1}; speed 25 m s^{-1}

2 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} -2 \\ -3 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -6 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 0 \\ r \end{pmatrix}$ respectively.
Given that A, B, C are collinear, find r.

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} -2 \\ 3 \\ r - -2 \end{pmatrix}$
$\overrightarrow{AC} = \lambda\overrightarrow{AB}$: component 1 gives $-2 = \lambda \times 2$, so $\lambda = -1$
$r - -2 = -1 \times 1$, so $r = -3$
Answer: $r = -3$

3 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

P(A) = 0.24 + 0.16 = 0.4 and $P(A \cap B) = 0.16$
Independent: $P(A \cap B) = P(A)P(B)$, so $P(B) = 0.16 \div 0.4 = 0.4$
$x = P(B) - P(A \cap B) = 0.4 - 0.16 = 0.24$
$y = 1 - (0.24 + 0.16 + 0.24) = 0.36$
Answer: x = 0.24, y = 0.36

4 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $4x^2 - 18x + 20 \geq 0$.

Factorise: $2(x - 2)(2x - 5) \geq 0$
Critical values: $x = 2$ and $x = \frac{5}{2}$
Sketch the ∪-shaped parabola: $x \leq 2$ or $x \geq \frac{5}{2}$
Answer: $x \leq 2$ or $x \geq \frac{5}{2}$

5 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -10 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-10 + 3\lambda = 2 + 3\mu$
j: $3 + 0\lambda = 2 + (-1)\mu$
Solving i and j: $\lambda = 3$, $\mu = -1$
k check: $-2 + 0(3) = -2$ and $0 + 2(-1) = -2$, equal
Point: $\begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-1} \\ \mathbf{3} \\ \mathbf{-2} \end{pmatrix}$

6 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:
(a) y = f(2x) (b) y = f(−x)

(a) y = f(2x): A(0, 2) maps to (0, 2), B(4, −4) maps to (2, −4)
(b) y = f(−x): A(0, 2) maps to (0, 2), B(4, −4) maps to (−4, −4)
Answer: (a) (0, 2), (2, −4) (b) (0, 2), (−4, −4)

7 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (2t^2 + 5)\mathbf{i} + (2t - 8)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of **i**, and its speed then.

Parallel to **i**: **j** component = 0
$2t - 8 = 0$, so $t = 4$
$\mathbf{v} = (2 \times 4^2 + 5)\mathbf{i} = 37\mathbf{i}$
Answer: t = 4 s, speed 37 m s^{-1}

8 Fixed-point iteration (All boards)

(a) Show that $x^2 - 5x - 3 = 0$ can be written as $x = \sqrt{5x + 3}$.
(b) Use $x_{n+1} = \sqrt{5x_n + 3}$ with $x_0 = 3$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^2 = 5x + 3$, so $x = \sqrt{5x + 3}$
(b) $x_1 = 4.2426$
$x_2 = 4.9207$
$x_3 = 5.2539$
Answer: $x_1 = 4.2426$, $x_2 = 4.9207$, $x_3 = 5.2539$

9 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $3^{2x} - 36 \times 3^x + 243 = 0$.

Let $y = 3^x$. Then $3^{2x} = y^2$, so $y^2 - 36y + 243 = 0$
$(y - 27)(y - 9) = 0$
$y = 27$ or $y = 9$
$3^x = 27$ gives $x = 3$; $3^x = 9$ gives $x = 2$.
Answer: x = 2 or x = 3

10 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.5. It is tossed 40 times and lands heads 13 times.
Test, at the 5% significance level, whether the probability of heads is different from 0.5.

H_0: p = 0.5, H_1: p ≠ 0.5. Under H_0, X ~ B(40, 0.5)

P(X ≤ 13) = 0.0192

Two-tailed: compare with 0.025. 0.0192 < 0.025, so reject H_0

There is sufficient evidence that the probability of heads is different from 0.5

Answer: reject H_0: sufficient evidence that the probability of heads is different from 0.5