Day 338 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has area 157.5 cm^2 and angle 1.4 radians.
Find (a) the radius, (b) the perimeter of the sector.

$\frac{1}{2}r^2\theta = 157.5$, so $r^2 = \frac{2 \times 157.5}{1.4} = 225$, $r = 15$

Perimeter $= 2r + r\theta = 30 + 21 = 51$ cm

Answer: (a) 15 cm (b) 51 cm

2 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trolley of mass 10 kg is moving on a smooth horizontal surface at 4 m s^{-1} when a constant horizontal force of 10 N, in the direction of motion, acts for 7 s.
Find the speed of the trolley at the end of this time and the distance it moves.

$F = ma$: $a = 10 \div 10 = 1$

$v = u + at = 4 + 1 \times 7$

$s = ut + \frac{1}{2}at^2$

Answer: 11 m s^{-1}; 52.5 m

3 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 10 pairs of values, $\bar{x} = 10$, $\bar{y} = 18$, $S_{xx} = 80$ and $S_{xy} = -160$.
(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Estimate y when $x = 8$.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{-160}{80} = -2$

$a = \bar{y} - b\bar{x} = 18 - -2 \times 10 = 38$

$y = 38 - 2x$

(b) $y = 38 + (-2) \times 8 = 22$

Answer: $y = 38 - 2x$; $y = 22$

4 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find y in terms of x, given that $\frac{dy}{dx} = \frac{5x}{y}$

and $y = 2$ when $x = 0$.

$y\,dy = 5x\,dx$, so $\frac{y^2}{2} = \frac{5x^2}{2} + c$

$x = 0$, $y = 2$: $c = 2$

$y^2 = 5x^2 + 4$, so $y = \sqrt{5x^2 + 4}$ $(y > 0)$

Answer: $y = \sqrt{5x^2 + 4}$

5 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^3 + 5n + 11$ is odd for all integers n.

$n^3 + 5n = n(n^2 + 5)$

If n is even, $n(n^2 + 5)$ is even.

If n is odd, n^2 is odd and $n^2 + 5$ is even, so $n(n^2 + 5)$ is even.

So $n^3 + 5n$ is always even, and adding 11 gives an odd number.

Answer: expression is odd

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6 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3\tan^2 x + 7\sec x + 7 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $3\sec^2 x + 7\sec x + 4 = 0$

$(3\sec x + 4)(\sec x + 1) = 0$, so $\sec x = -\frac{4}{3}$ or -1

$\sec x = -\frac{4}{3}$, so $\cos x = -\frac{3}{4}$: $x = 138.6°, 221.4°$

$\sec x = -1$, so $\cos x = -1$: $x = 180.0°$

Answer: $x = 138.6°, 180.0°, 221.4°$

7 Binomial: smallest n (All boards)

Given that each attempt at a task succeeds independently with probability 0.15, find the least number of attempts needed for the probability of at least one success to exceed 0.99.

$P(\text{at least one}) = 1 - 0.85^n > 0.99$

$0.85^n < 0.01$

$n > \frac{\log 0.01}{\log 0.85} = 28.34$

Answer: $n = 29$

8 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a square number cannot be written in the form $3k + 2$, where k is an integer.

Write the integer as $n = 3q + j$, where $j = 0, 1, ..., 2$.

n^2 leaves the same remainder as j^2 when divided by 3.

$j = 0, 1, 2$ give remainders 0, 1, 1.

The only possible remainders are 0, 1, never 2.

Answer: possible remainders 0, 1, never 2

9 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 9 kg lies on a rough horizontal table (coefficient of friction 0.2). It is joined by a light string over a smooth pulley to a particle of mass 9 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.2 \times 9g = 9a$

Hanging: $9g - T = 9a$

Add: $9g - 1.8g = 18a$

$a = 3.92$

$T = 9(9.8 - 3.92)$

Answer: $a = 3.92$ m s^{-2}; $T = 52.92$ N

10 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + px^2 + qx + 8$.

When f(x) is divided by $(x + 1)$ the remainder is 12, and when divided by $(x + 3)$ the remainder is 8. Find p and q.

$f(-1) = 12$: $-1 + p - q + 8 = 12$, so $p - q = 5$

$f(-3) = 8$: $-27 + 9p - 3q + 8 = 8$, so $9p - 3q = 27$

Solving simultaneously: $p = 2$, $q = -3$

Answer: $p = 2$, $q = -3$