Day 337 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) y = f(2x)
(b) y = f(−x)

(a) y = f(2x) is a stretch, scale factor $\frac{1}{2}$, parallel to the x-axis.

(b) y = f(−x) is a reflection in the y-axis.

Answer: (a) stretch, scale factor $\frac{1}{2}$, parallel to the x-axis (b) reflection in the y-axis

2 Trig equations in radians (All boards)

Solve cos x = −0.3 for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

cos θ = −0.3 for $0 \leq x \leq 2\pi$
Calculator: x = 1.875, then use the symmetry of the graph
x = 1.875, 4.408
Answer: x = 1.88, 4.41

3 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 8 m s^{-1} brakes and comes to rest with constant deceleration 1.6 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 8^2 - 2 \times 1.6 \times s$
$s = 64 \div 3.2 = 20$
(b) $0 = 8 - 1.6\,t$, so $t = 8 \div 1.6$
Answer: (a) 20 m (b) 5 s

4 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $4x^2y + 3y^3 - xy = 6$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at (1, 1).

Differentiate with respect to x: $4x^2\frac{dy}{dx} + 8xy + 9y^2\frac{dy}{dx} - x\frac{dy}{dx} - y = 0$

$$\frac{dy}{dx} = \frac{y(1 - 8x)}{4x^2 + 9y^2 - x}$$

At (1, 1): gradient = $-\frac{7}{12}$

Answer: $\frac{dy}{dx} = \frac{y(1 - 8x)}{4x^2 + 9y^2 - x}$; gradient $-\frac{7}{12}$

5 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ab^x$, where a and b are constants.
A plot of $\log_{10} y$ against x is a straight line with intercept 0.903 and gradient 0.176.
Find a and b to 2 significant figures.

$\log_{10} y = \log_{10} a + x \log_{10} b$
$\log_{10} a = 0.903$, so $a = 10^{0.903} = 8.0$
$\log_{10} b = 0.176$, so $b = 10^{0.176} = 1.5$
Answer: a = 8.0, b = 1.5

6 Car towing a trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1500 kg tows a trailer of mass 500 kg along a straight horizontal road by a light horizontal tow bar. The resistances are 500 N on the car and 200 N on the trailer.
The engine's driving force is 1500 N. Find the acceleration and the tension in the tow bar.

Whole system: $1500 - 500 - 200 = 2000\,a$
$a = 0.4$
Trailer: $T - 200 = 500 \times 0.4$
Answer: $a = 0.4$ m s^{-2}; $T = 400$ N

7 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + x)^{\frac{1}{3}}$ up to the x^2 term.
(b) Use $x = 0.03$ to estimate $\sqrt[3]{1.03}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = \frac{1}{3}$, $a = 1$:

$= 1 + \frac{1}{3}x - \frac{1}{9}x^2$

(b) $1 + 1 \times 0.03 = 1.03$, so substitute $x = 0.03$:

$1 + \frac{1}{3} \times 0.03 - \frac{1}{9} \times 0.03^2 = 1.00990$

Answer: (a) $1 + \frac{1}{3}x - \frac{1}{9}x^2 + \ldots$ (b) 1.00990

8 Binomial two-tailed test (All boards)

A factory says a proportion 0.4 of its parts are defective. In a sample of 50 parts, 26 are defective.
Test, at the 5% significance level, whether the proportion is different from 0.4.

H_0: $p = 0.4$, H_1: $p \neq 0.4$. Under H_0, $X \sim B(50, 0.4)$
$P(X \geq 26) = 1 - P(X \leq 25) = 0.0573$
Two-tailed: compare with 0.025. $0.0573 > 0.025$, so do not reject H_0
There is insufficient evidence that the proportion is different from 0.4
Answer: do not reject H_0: insufficient evidence that the proportion is different from 0.4

9 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x - 4y = k$ is a tangent to the circle
$(x + 1)^2 + (y + 3)^2 = 25$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(3, -4)$.
A unit step in this direction is $(3, -4)/5$, so the points of contact are
$(-1, -3) \pm 5 \times (3, -4)/5 = (-1, -3) \pm (3, -4)$
T = $(2, -7)$ or $(-4, 1)$, and $k = 3x - 4y$ gives:
$k = 34$ or $k = -16$
Answer: $k = 34$ or $k = -16$

10 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), find an approximation of the
form $1 + p\theta + q\theta^2$ for
$\sin 4\theta + \cos 2\theta + \tan \theta$.

$\sin 4\theta \approx 4\theta$, $\cos 2\theta \approx 1 - \frac{4\theta^2}{2}$, $\tan \theta \approx \theta$

Sum $\approx 1 + 4\theta - \frac{4\theta^2}{2} + \theta$

$\approx 1 + 5\theta - 2\theta^2$
Answer: $p = 5$, $q = -2$