Day 336 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Conditional probability from a count (All boards)

80 customers: 14 buy coffee, 22 buy tea, 51 do neither.
(a) How many do both?
(b) Given a person who buys tea, find P(this person buys coffee).

(a) At least one: $80 - 51 = 29$. Both $= 14 + 22 - 29 = 7$

(b) Of the 22 who buy tea, 7 also do the other: $P = \frac{7}{22} = \frac{7}{22}$

Answer: (a) 7 (b) $\frac{7}{22}$

2 Locating roots: change of sign (All boards)

The function $f(x) = x^3 - 4x - 11$.
Show that $f(x) = 0$ has a root between $x = 2.5$ and $x = 3$.

$f(2.5) = -5.38$
$f(3) = 4.00$
f is continuous and changes sign, so there is a root in the interval
Answer: f(2.5) < 0 and f(3) > 0: sign change

3 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has $E(X) = 4$ and $Var(X) = 5$.
Find (a) $E(4X + 5)$,
(b) $Var(4X + 5)$, (c) $E(X^2)$.

(a) $E(aX + b) = aE(X) + b = 4 \times 4 + 5 = 21$
(b) $Var(aX + b) = a^2Var(X) = 16 \times 5 = 80$
(c) $Var(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 5 + 16 = 21$
Answer: (a) 21 (b) 80 (c) 21

4 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{6}{2 + 2\sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are integers.

Multiply top and bottom by the conjugate $(2 - 2\sqrt{2})$:
Denominator: $(2 + 2\sqrt{2})(2 - 2\sqrt{2}) = 2^2 - 2^2 \times 2 = -4$
Numerator: $(6)(2 - 2\sqrt{2}) = 12 - 12\sqrt{2}$
So the fraction $= \frac{12 - 12\sqrt{2}}{-4} = -3 + 3\sqrt{2}$

Answer: $-3 + 3\sqrt{2}$

5 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}-1\\4\\-1\end{pmatrix}$ and B has position vector $\begin{pmatrix}5\\13\\-10\end{pmatrix}$.

The point P lies on AB with AP : PB = 2 : 1.

Find the position vector of P.

$\text{AB} = \begin{pmatrix}6\\9\\-9\end{pmatrix}$

$\text{AP} = \frac{2}{3}\text{AB} = \begin{pmatrix}4\\6\\-6\end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix}3\\10\\-7\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{3}\\\mathbf{10}\\\mathbf{-7}\end{pmatrix}$

6 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 7 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{5}{12}$ and $\tan\beta = \frac{3}{4}$.

Take g = 9.8 m s^{-2}.

Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{12}{13} = T_2 \times \frac{4}{5}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 68.6$

From the first: $T_1 = \frac{13}{15} T_2$; into the second: $T_2 \times \frac{14}{15} = 68.6$

Answer: T_1 = 63.7 N (at α); T_2 = 73.5 N (at β)

7 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $25^{x-4} = 5^{-2x+5}$.

Write both sides as powers of 5: $5^{2(x-4)} = 5^{-2x+5}$

Equate the indices: $2x - 8 = -2x + 5$

$4x = 13$

$x = \frac{13}{4}$

Answer: $x = \frac{13}{4}$

8 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected from level ground with horizontal velocity component 39 m s^{-1} and vertical velocity component 44.1 m s^{-1} upwards.

Take g = 9.8 m s^{-2}.

Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 44.1\,t - 4.9t^2$, so $t = 44.1 \div 4.9$

(b) Range = horizontal speed × time = 39 × 9

Answer: (a) 9 s (b) 351 m

9 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that 9 cos θ + 12 sin θ ≡ 15 cos(θ − 53.1°),
solve 9 cos θ + 12 sin θ = 0.5 for 0 ≤ θ ≤ 360°. (1 d.p.)

15 cos(θ − 53.1°) = 0.5, so $\cos(\theta - 53.1°) = \frac{0.5}{15}$

θ − 53.1° = ±88.1°
θ = 53.1° + 88.1° = 141.2°
θ = 53.1° − 88.1° = −35.0°, so θ = 325.0°
Answer: θ = 141.2°, 325.0°

10 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 4x + 3y = k is a tangent to the circle
$(x - 4)^2 + (y - 3)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (4, 3).
A unit step in this direction is (4, 3)/5, so the points of contact are
(4, 3) ± 10 × (4, 3)/5 = (4, 3) ± (8, 6)
T = (12, 9) or (−4, −3), and k = 4x + 3y gives:
k = 75 or k = −25
Answer: k = 75 or k = −25