Day 335 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has area 101.25 cm^2 and angle 0.9 radians.
Find (a) the radius, (b) the perimeter of the sector.

$\frac{1}{2}r^2\theta = 101.25$, so $r^2 = \frac{2 \times 101.25}{0.9} = 225$, $r = 15$

Perimeter $= 2r + r\theta = 30 + 13.5 = 43.5$ cm

Answer: (a) 15 cm (b) 43.5 cm

2 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 36 kW. The resistance to motion is constant at 600 N.

Find the acceleration when the speed is 20 m s^{-1}.

$P = Fv$: $F = 36000 \div 20 = 1800$ N

$F - R = ma$: $1800 - 600 = 1000\,a$

Answer: 1.2 m s^{-2}

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (3, 6) and (−5, 7).
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{3 + (-5)}{2}, \frac{6 + 7}{2}\right) = \left(-1, \frac{13}{2}\right)$

(b) Gradient $= \frac{1}{-8} = -\frac{1}{8}$

$y - 6 = -\frac{1}{8}(x - 3)$

Multiply out and collect terms: $x + 8y - 51 = 0$

Answer: (a) $\left(-1, \frac{13}{2}\right)$ (b) $x + 8y - 51 = 0$

4 Standardising to compare (All boards)

Marks are normal: Maths N(55, 10^2), Physics N(65, 12^2).
Anna scores 50 in Maths and 74 in Physics. Which is her better performance, relatively?

z-score in Maths $= \frac{50 - 55}{10} = -0.500$

z-score in Physics $= \frac{74 - 65}{12} = 0.750$

The higher z-score is in Physics

Answer: Physics

5 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{4}{5}$ and $\cos B = \frac{8}{17}$, where A and B are acute,

find the exact value of $\sin(A + B)$.

$\cos A = \frac{3}{5}$ and $\sin B = \frac{15}{17}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$= \frac{4}{5} \times \frac{8}{17} + \frac{3}{5} \times \frac{15}{17} = \frac{77}{85}$

Answer: $\frac{77}{85}$

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6 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 3x^2 + 18x + 34$ for $x \geq -3$.
Find $f^{-1}(x)$ and state its domain.

Complete the square: $f(x) = 3(x + 3)^2 + 7$

$y = 3(x + 3)^2 + 7$, so $(x + 3)^2 = \frac{y - 7}{3}$

As $x + 3 \geq 0$ take the positive root: $x = -3 + \sqrt{\frac{y - 7}{3}}$

As $x + 3 \geq 0$, take the positive root: $x = -3 + \sqrt{\frac{y - 7}{3}}$

Answer: $f^{-1}(x) = -3 + \sqrt{\frac{x - 7}{3}}$, domain $x \geq 7$

7 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = t^3 + 3t^2$ metres.
Find (a) the average velocity over the first 4 seconds, (b) the velocity at t = 4.

(a) $s(4) = 112$, so average velocity $= 112 \div 4$
(b) $v = ds/dt = 3t^2 + 6t$, $v(4) = 72$
Answer: (a) 28 m s^{-1} (b) 72 m s^{-1}

8 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of

$(3x + \frac{2}{x})^6$.

The general term is $C(6, k)(3x)^{6-k}(2/x)^k$, with power of x equal to $6 - 2k$
Independent of x: $6 - 2k = 0$, so $k = 3$
Term $= 20 \times 3^3 \times 2^3 = 4320$
Answer: 4320

9 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $4\cos^2 x - \sin x - 4 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $-4\sin^2 x - \sin x = 0$, i.e. $4\sin^2 x + \sin x = 0$

$(4 \sin x + 1)(\sin x) = 0$, so $\sin x = -\frac{1}{4}$ or 0

$\sin x = -\frac{1}{4}$: $x = 194.5°, 345.5°$

$\sin x = 0$: $x = 0.0°, 180.0°, 360.0°$
Answer: x = 0.0°, 180.0°, 194.5°, 345.5°, 360.0°

10 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
$P(X < 40) = 0.1587$ and $P(X > 60) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$
$40 = \mu + (-0.9998)\sigma$ and $60 = \mu + 0.9998\sigma$
Subtract: $20 = 2.000\ \sigma$, so $\sigma = 10.0$
$\mu = 40 - (-0.9998)(10.00) = 50.0$
Answer: $\mu = 50.0$, $\sigma = 10.0$