Day 334 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + 6x^2 + kx$.
Given that $(x - 1)$ is a factor of $f(x)$,
(a) find the value of k,
(b) factorise $f(x)$ completely.

$f(1) = 0$: $2 + 6 + k = 0$
$k = -8$
$f(x) = (x - 1)(2x^2 + 8x)$
$= 2(x - 1)x(x + 4)$
Answer: (a) $k = -8$ (b) $f(x) = 2(x - 1)x(x + 4)$

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 209; systematic sample of size 19.
(a) Find the sampling interval.
(b) The first item is number 3. Find the 6-th item.

(a) Interval $= 209 \div 19 = 11$
(b) 6-th item $= 3 + 5 \times 11 = 58$
Answer: interval 11; item number 58

3 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{2}{x - 4} - 3$.
(a) Write down the asymptotes.
(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = 4$, and $y \to -3$ as $x \to \pm\infty$
Asymptotes: $x = 4$ and $y = -3$
(b) $x = 0$: $y = -\frac{1}{2} - 3 = -\frac{7}{2}$
$y = 0$: $\frac{2}{x - 4} = 3$, so $x - 4 = \frac{2}{3}$, $x = \frac{14}{3}$
Answer: (a) $x = 4$, $y = -3$ (b) $(0, -\frac{7}{2})$ and $(\frac{14}{3}, 0)$

4 Beam on two supports: reactions (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform beam AB of length 10 m and mass 40 kg rests horizontally on supports at C and D, where AC = 1 m and BD = 1 m.
A particle of mass 15 kg is placed on the beam 3 m from B. Take $g = 9.8$ m s^{-2}.
Find the reactions at C and D.

Moments about C: $R_D \times 8 = 40\,g \times 4 + 15\,g \times 6$
$R_D = 2450 \div 8$
Resolve vertically: $R_C + R_D = 55\,g$
Answer: $R_C = 232.75$ N; $R_D = 306.25$ N

5 Grouped data: mean and median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises 54 measurements of d.
(a) Estimate the mean.
(b) Estimate the median using linear interpolation.

(a) Using mid-points: $\Sigma fx = 1500$, so mean $= 1500 \div 54 = 27.8$
(b) $\frac{n}{2} = 27$. Cumulative frequency before the median class = 14
Median class 20 to 30. Median $= 20 + \frac{13}{14} \times 10 = 29.3$
Answer: mean ≈ 27.8, median ≈ 29.3

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6 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = 2t + 1$, $y = t^2 - 4$.
Find $\frac{dy}{dx}$ in terms of t, and the equation of the tangent at $t = -1$.

$\frac{dx}{dt} = 2$, $\frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = t$; at $t = -1$ it is -1

Point: $(-1, -3)$
$y + 3 = -1(x + 1)$, so $y = -x - 4$
Answer: $\frac{dy}{dx} = t$; tangent $y = -x - 4$

7 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 50 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 10 to t = 40.
The floors are 720 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (50 + 30) × V = 720
80 V = 1440 so V = 18
Deceleration = 18 ÷ 10
Answer: V = 18; deceleration 1.8 m s^{-2}

8 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $3\cos\theta + 2\sin\theta \equiv \sqrt{13}\cos(\theta - 33.7°)$,
solve $3\cos\theta + 2\sin\theta = -1$ for $0 \le \theta \le 360°$. (1 d.p.)

$\sqrt{13}\cos(\theta - 33.7°) = -1$, so $\cos(\theta - 33.7°) = \frac{-1}{\sqrt{13}}$

$\theta - 33.7° = \pm 106.1°$
$\theta = 33.7° + 106.1° = 139.8°$
$\theta = 33.7° - 106.1° = -72.4°$, so $\theta = 287.6°$
Answer: θ = 139.8°, 287.6°

9 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x - 5)(x - 1)(x - 4) > 0$.

Critical values: $x = 1$, $x = 4$, $x = 5$
The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.
Reading from the sketch: $1 < x < 4$ or $x > 5$
Answer: $1 < x < 4$ or $x > 5$

10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -4 \\ -5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-4 + 2\lambda = -2 + 3\mu$

j: $-5 + 2\lambda = -3 + 2\mu$

Solving i and j: $\lambda = 1$, $\mu = 0$

k check: $2 + (-1)(1) = 1$ and $1 + 0(0) = 1$, equal

Point: $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-2} \\ \mathbf{-3} \\ \mathbf{1} \end{pmatrix}$