Day 333 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^2 - 11x \leq -12$.

Rearrange: $2x^2 - 11x + 12 \leq 0$
Factorise: $(x - 4)(2x - 3) \leq 0$

Critical values: $x = \frac{3}{2}$ and $x = 4$

Sketch the ∪-shaped parabola: $\frac{3}{2} \leq x \leq 4$

Answer: $\frac{3}{2} \leq x \leq 4$

2 Inverse normal (All boards)

The mass X g of a flour bag is normal with mean 45, s.d. 0.5.
Find k (3 s.f.): (a) P(X < k) = 0.05,
(b) P(X > k) = 0.1.

(a) $z = -1.645$, so $k = 45 + (-1.645) \times 0.5 = 44.2$
(b) P(X < k) = 0.9, $z = 1.282$, so $k = 45 + 1.282 \times 0.5 = 45.6$
Answer: (a) 44.2 (b) 45.6

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 8 cm and angle 90°.
Find, in terms of π, the arc length and the area of the sector.

$\theta = 90° = \frac{\pi}{2}$ radians

Arc length $= r\theta = 4\pi$ cm

Area $= \frac{1}{2}r^2\theta = 16\pi$ cm^2

Answer: arc 4π cm, area 16π cm^2

4 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 4t^3 - 2t^2 - t$ metres.
Find its velocity and acceleration when t = 4.

$v = ds/dt = 12t^2 - 4t - 1$
$a = dv/dt = 24t - 4$
$t = 4$: $v = 175$, $a = 92$
Answer: $v = 175$ m s^{-1}, $a = 92$ m s^{-2}

5 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{-1}{x - 2} - 3$.

(a) Write down the asymptotes.
(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = 2$, and $y \to -3$ as $x \to \pm\infty$
Asymptotes: $x = 2$ and $y = -3$

(b) $x = 0$: $y = \frac{1}{2} - 3 = -\frac{5}{2}$

$y = 0$: $\frac{-1}{x - 2} = 3$, so $x - 2 = -\frac{1}{3}$, $x = \frac{5}{3}$

Answer: (a) $x = 2$, $y = -3$ (b) $(0, -\frac{5}{2})$ and $(\frac{5}{3}, 0)$

6 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 10, frequency 24, is a bar 5 cm wide, 2.4 cm tall.
(a) Find the bar height for width 10, frequency 30.
(b) Find the frequency of a 1 cm by 2 cm bar.

Area of bar $\propto$ frequency: $5 \times 2.4 = 12$ cm^2 represents 24
so 1 unit of frequency = 0.5 cm^2
(a) Width = 5 cm, area = $0.5 \times 30 = 15$ cm^2, height = $15 \div 5 = 3$ cm
(b) Area = $1 \times 2 = 2$ cm^2, frequency = $2 \div 0.5 = 4$
Answer: (a) 3 cm (b) 4

7 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 500 and common ratio 0.5.
Find the smallest n for which the nth term is less than 0.01.

$500 \times 0.5^{n-1} < 0.01$
$(n - 1) \lg 0.5 < \lg(0.01 \div 500)$
lg 0.5 is negative, so dividing reverses the inequality:
$n - 1 > \lg(0.01 \div 500) \div \lg 0.5 = 15.610$, so $n > 16.610$
The smallest integer n is 17
Answer: n = 17

8 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 2e^x - 15 = 0$,
giving any answers as exact values.

Let $y = e^x$: $y^2 - 2y - 15 = 0$
$(y + 3)(y - 5) = 0$
$e^x > 0$, so $y = -3$ is rejected; $e^x = 5$
$x = \ln 5$
Answer: x = ln 5

9 Beam on two supports: reactions (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform beam AB of length 6 m and mass 60 kg rests horizontally on supports at C and D, where AC = 2 m and BD = 1.5 m.

A particle of mass 5 kg is placed on the beam 1.5 m from B. Take g = 9.8 m s^{-2}.
Find the reactions at C and D.

Moments about C: $R_D \times 2.5 = 60\,g \times 1 + 5\,g \times 2.5$
$R_D = 710.5 \div 2.5$
Resolve vertically: $R_C + R_D = 65\,g$
Answer: R_C = 352.8 N; R_D = 284.2 N

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10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 8 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $2 + (-2)\lambda = 8 + 3\mu$

j: $-2 + 3\lambda = 4 + 3\mu$

Solving i and j: $\lambda = 0$, $\mu = -2$

k check: $1 + (-2)(0) = 1$ and $-1 + (-1)(-2) = 1$, equal

Point: $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{2} \\ \mathbf{-2} \\ \mathbf{1} \end{pmatrix}$