Day 332 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A survey covers 56 urban, 44 suburban and 24 rural households.
A stratified sample of size 31 is taken. Find the number selected from each group.

Total = 56 + 44 + 24 = 124

Sampling fraction = 31/124 = $\frac{1}{4}$

urban: 56 ÷ 4 = 14
suburban: 44 ÷ 4 = 11
rural: 24 ÷ 4 = 6
Answer: urban 14, suburban 11, rural 6

2 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\sqrt{45} + \sqrt{245} + \sqrt{180}$, giving your answer in the form $k\sqrt{5}$.

$\sqrt{45} = 3\sqrt{5}$, $\sqrt{245} = 7\sqrt{5}$, $\sqrt{180} = 6\sqrt{5}$
$3\sqrt{5} + 7\sqrt{5} + 6\sqrt{5} = 16\sqrt{5}$
Answer: $16\sqrt{5}$

3 Independent and mutually exclusive events (All boards)

A and B are mutually exclusive events with P(A) = 0.1 and P(A ∪ B) = 0.7.
(a) Find P(B).
(b) Find P(A' ∩ B').

(a) Mutually exclusive: P(A ∪ B) = P(A) + P(B)
P(B) = 0.7 − 0.1 = 0.6
(b) P(A' ∩ B') = 1 − P(A ∪ B) = 0.3
Answer: (a) 0.6 (b) 0.3

4 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when
$-x^3 - 7x^2 - 13x - 10$ is divided by $(x + 3)$.

$-x^2 \times (x + 3) = -x^3 - 3x^2$, leaving $-4x^2 - 13x - 10$
$-4x \times (x + 3) = -4x^2 - 12x$, leaving $-x - 10$
$-1 \times (x + 3) = -x - 3$, leaving -7
Quotient $-x^2 - 4x - 1$, remainder -7
Answer: quotient $-x^2 - 4x - 1$, remainder -7

5 Exponential growth and decay models (All boards)

A population is modelled by $N = N_0 e^{kt}$, where t is in hours. Initially N = 1500 and after 2 hours N = 3750.
Find (a) k to 3 s.f., (b) when N reaches 7500, to 3 s.f.

$N_0 = 1500$, so $3750 = 1500\, e^{2k}$, $e^{2k} = 2.5$

(a) $k = \frac{1}{2} \ln 2.5 = 0.4581$

(b) $1500\, e^{kt} = 7500$, so $t = \frac{\ln 5}{k} = 3.513$ hours

Answer: (a) k = 0.458 (b) t = 3.51 hours

6 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $3t^2 - 21t + 30$ m s^{-1}.
Find the first time P is at instantaneous rest, and its distance from O then.

v = 0: $3(t - 2)(t - 5) = 0$, so t = 2 or t = 5
First time t = 2

$s = \int_0^2 v\, dt = [\, t^3 - \frac{21}{2}t^2 + 30t \,]$ from 0 to 2

Answer: t = 2 s; 26 m

Day 332 **A Level Maths: B Tier 10-a-day: Model Solutions**

7 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations

to find the value of $\lim_{\theta \to 0} \frac{\tan 2\theta \sin 3\theta}{1 - \cos 2\theta}$.

$\tan 2\theta \sin 3\theta \approx 2\theta \times 3\theta = 6\theta^2$

$1 - \cos 2\theta \approx \frac{4\theta^2}{2}$

Ratio = $\frac{6\theta^2}{\frac{4\theta^2}{2}} = 3$

Answer: 3

8 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 5x)^{\frac{1}{2}}$ up to the x^2 term.

(b) Use x = 0.04 to estimate $\sqrt{1.20}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = \frac{1}{2}$, a = 5:

$= 1 + \frac{5}{2}x - \frac{25}{8}x^2$

(b) 1 + 5 × 0.04 = 1.20, so substitute x = 0.04:

$1 + \frac{5}{2} \times 0.04 - \frac{25}{8} \times 0.04^2 = 1.09500$

Answer: (a) $1 + \frac{5}{2}x - \frac{25}{8}x^2 + \ldots$ (b) 1.09500

9 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 25°C: $\frac{dT}{dt} = -k(T - 25)$. Initially T = 100 and after 8 minutes T = 45.

Find k to 3 s.f. and the time to cool to 35°C, to 3 s.f.

$\frac{1}{T - 25}\, dT = -k\, dt$, so $\ln(T - 25) = -kt + c$, i.e. $T = 25 + 75e^{-kt}$

t = 8: $45 = 25 + 75e^{-8k}$, so $e^{-8k} = \frac{4}{15}$

$k = \frac{1}{8} \ln \frac{75}{20} = 0.1652$

$35 = 25 + 75e^{-kt}$: $t = \frac{1}{k} \ln \frac{75}{10} = 12.20$

Answer: k = 0.165, t = 12.2 minutes

10 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A missile is projected horizontally at 117.6 m s^{-1} from a point high above the ground.

Take g = 9.8 m s^{-2}.

Find the speed and the direction of motion 16 s later, to 1 d.p. below the horizontal.

Vertical speed = 9.8 × 16 = 156.8

Speed = $\sqrt{(117.6^2 + 156.8^2)} = 196$

$\tan \alpha = 156.8 \div 117.6$, $\alpha = 53.1°$

Answer: 196 m s^{-1} at 53.1° below horizontal