Day 331 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(8\mathbf{i} - 3\mathbf{j})$ N and $(4\mathbf{i} - 6\mathbf{j})$ N act on a particle of mass 3 kg.
Find the magnitude of the acceleration of the particle.

Resultant = $(8\mathbf{i} - 3\mathbf{j}) + (4\mathbf{i} - 6\mathbf{j}) = 12\mathbf{i} - 9\mathbf{j}$
$|R| = \sqrt{(12^2 + -9^2)} = 15$ N
$a = F/m = 15 \div 3$
Answer: 5 m s^{-2}

2 Basic trig equations (degrees) (All boards)

Solve $\tan 2x = -3.1$ for $0 \leq x \leq 360°$, giving answers to 1 d.p.

Let $\theta = 2x$, so $\tan \theta = -3.1$ with θ in range 0° to 720°
$\theta = 107.88°, 287.88°, 467.88°, 647.88°$
$x = 53.9°, 143.9°, 233.9°, 323.9°$
Answer: x = 53.9°, 143.9°, 233.9°, 323.9°

3 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 + 5x + 4$ and the line $y = 5x + 5$ meet at the points A and B.
Find the coordinates of A and B.

$x^2 + 5x + 4 = 5x + 5$
$x^2 - 1 = 0$, so $(x + 1)(x - 1) = 0$
$x = -1$ or $x = 1$
$y = 5x + 5$: $y = 0$ and $y = 10$
Answer: A(−1, 0), B(1, 10)

4 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}5\\0\end{pmatrix}$ and B has position vector $\begin{pmatrix}-1\\6\end{pmatrix}$.

The point P lies on AB with AP : PB = 2 : 1.
Find the position vector of P.

$\text{AB} = \begin{pmatrix}-6\\6\end{pmatrix}$
$\text{AP} = \frac{2}{3}\text{AB} = \begin{pmatrix}-4\\4\end{pmatrix}$
$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix}1\\4\end{pmatrix}$
Answer: $\begin{pmatrix}\mathbf{1}\\\mathbf{4}\end{pmatrix}$

5 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 5 kg is pulled across smooth horizontal ice by a rope with tension 39 N, inclined at α above the horizontal, where $\tan \alpha = \frac{5}{12}$.

Take $g = 9.8$ m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos \alpha = 12/13$, $\sin \alpha = 5/13$
(a) Horizontal: $39 \times 12/13 = 5a$
(b) Vertical: $R + 39 \times 5/13 = 5 \times 9.8$
Answer: (a) 7.2 m s^{-2} (b) 34 N

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6 Binomial: smallest n (All boards)

Given that each lottery ticket wins a prize independently with probability 0.05, find the least number of tickets needed for the probability of at least one success to exceed 0.95.

P(at least one) = $1 - 0.95^n > 0.95$

$0.95^n < 0.05$

$n > \frac{\log 0.05}{\log 0.95} = 58.40$

Answer: n = 59

7 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\frac{4}{\sqrt{2}} + \sqrt{18}$, giving your answer in the form $k\sqrt{2}$.

$\frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

$\sqrt{18} = 3\sqrt{2}$

$2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

Answer: $5\sqrt{2}$

8 Binomial one-tailed test (All boards)

A manufacturer claims that a proportion 0.4 of its sweets are red. A sample of 15 sweets has 2 red. Test, at the 1% level, whether the true proportion is lower than claimed.

H_0: p = 0.4, H_1: p < 0.4. Let X ~ B(15, 0.4) under H_0

$P(X \leq 2) = 0.0271$

0.0271 > 0.01, so do not reject H_0

There is insufficient evidence that the proportion is lower than claimed

Answer: do not reject H_0: insufficient evidence that the proportion is lower

9 Trig equations in radians (All boards)

Solve $\sin 2x = 0.2$ for $0 \leq x \leq \pi$,
giving answers to 3 significant figures.

Let $\theta = 2x$, so $\sin \theta = 0.2$ for $0 \leq \theta \leq 2\pi$

Calculator: $\theta = 0.2014$, then use the symmetry of the graph

$\theta = 0.2014, 2.940$

$x = \frac{\theta}{2}$

x = 0.1007, 1.470

Answer: x = 0.101, 1.47

10 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $4x + 3y = k$ is a tangent to the circle
$(x + 3)^2 + (y + 3)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (4, 3).

A unit step in this direction is (4, 3)/5, so the points of contact are

$(-3, -3) \pm 10 \times (4, 3)/5 = (-3, -3) \pm (8, 6)$

T = (5, 3) or (−11, −9), and k = 4x + 3y gives:

k = 29 or k = −71

Answer: k = 29 or k = −71