Day 330 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Solving exponential equations using logarithms (All boards)

Solve $e^{-2x} = 25$, giving your answer to 3 s.f.

Take natural logs: $-2x = \ln 25$

$x = \frac{\ln 25}{-2} = -1.609$

Answer: $x = -1.61$

2 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A ship moves with constant velocity $(2\mathbf{i} - \mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(7\mathbf{j})$ m. Find (a) its position vector when t = 5, (b) the value of t when it crosses the line through O in the direction of $\mathbf{i}$.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (7\mathbf{j}) + 5(2\mathbf{i} - \mathbf{j}) = (10\mathbf{i} + 2\mathbf{j})$

(b) $\mathbf{j}$ component zero: $7 - t = 0$, so $t = 7$

Answer: (a) $(10\mathbf{i} + 2\mathbf{j})$ m (b) t = 7 s

3 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = x^2e^{2-2x} - 2$.

Find the equation of the tangent to C at the point where x = 1, in the form y = mx + c.

$\frac{dy}{dx} = -2x(x - 1)e^{2-2x}$

At x = 1: $y = -1$ and $\frac{dy}{dx} = 0$

The tangent is horizontal, so $y = -1$

Answer: $y = -1$

4 Poisson distribution (CIE)

X ~ Po(3), modelling the number of emails received per hour.

Find, to 3 significant figures,

(a) $P(X \geq 2)$

(b) $P(X = 2)$

(a) $P(X \geq 2) = 1 - P(X \leq 1) = 0.801$

(b) $P(X = 2) = e^{-3} \times \frac{3^2}{2!} = 0.224$

Answer: (a) 0.801 (b) 0.224

5 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{6 + \sqrt{6}}{7 - 3\sqrt{6}}$ in the form $p + q\sqrt{6}$, where p and q are integers.

Multiply top and bottom by the conjugate $(7 + 3\sqrt{6})$:

Denominator: $(7 - 3\sqrt{6})(7 + 3\sqrt{6}) = 7^2 - 3^2 \times 6 = -5$

Numerator: $(6 + \sqrt{6})(7 + 3\sqrt{6}) = 60 + 25\sqrt{6}$

So the fraction $= \frac{60 + 25\sqrt{6}}{-5} = -12 - 5\sqrt{6}$

Answer: $-12 - 5\sqrt{6}$

6 Binomial critical region (All boards)

X ~ B(50, p). H_0: p = 0.3 is to be tested against H_1: p < 0.3 at the 10% significance level.

Find the critical region and the actual significance level.

Under H_0, X ~ B(50, 0.3)

$P(X \leq 10) = 0.0789 \leq 0.1$

$P(X \leq 11) = 0.1390 > 0.1$

The critical region is $X \leq 10$

Answer: $X \leq 10$; actual significance level 7.89%

7 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $3t^2 - 30t + 72$ m s^{-1}.
Find the first time P is at instantaneous rest, and its distance from O then.

$v = 0$: $3(t - 4)(t - 6) = 0$, so $t = 4$ or $t = 6$
First time $t = 4$

$s = \int_0^4 v\,dt = [\,t^3 - 15t^2 + 72t\,]$ from 0 to 4

Answer: t = 4 s; 112 m

8 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 5 \\ 0 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $5 + 2\lambda = 4 + 1\mu$
j: $0 + 3\lambda = -5 + (-2)\mu$
Solving i and j: $\lambda = -1$, $\mu = -1$
k check: $-5 + (-2)(-1) = -3$ and $-5 + (-2)(-1) = -3$, equal

Point: $\begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{3} \\ \mathbf{-3} \\ \mathbf{-3} \end{pmatrix}$

9 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(1 - 4x)(1 - 3x)^5$.

$(1 - 3x)^5$ has x^1 coefficient -15 and x^2 coefficient 90
x^2 term of the product $= 1 \times 90 + (-4) \times (-15)$
$= 150$

Answer: 150

10 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(−3, −8) and B(5, −4), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:
$9 + (h + 8)^2 = 25 + (h + 4)^2$
Expanding and cancelling h^2: $16h + 73 = 8h + 41$
$h = -4$
$r^2 = CA^2 = 25$

Answer: centre (0, −4), $x^2 + (y + 4)^2 = 25$