Day 33 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A marble is projected vertically upwards from ground level with speed 39.2 m s^{-1}.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height reached, (b) the total time before it returns to the ground.

(a) At the top v = 0: $0 = 39.2^2 - 2 \times 9.8 \times H$
$H = 1536.64 \div 19.6 = 78.4$
(b) s = 0: $0 = 39.2\,t - 4.9t^2$, so $t = 39.2 \div 4.9$
Answer: (a) 78.4 m (b) 8 s

2 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Values of y = f(x) are shown.
x: 0, 0.5, 1, 1.5, 2
y: 1.8, 2.2, 1.9, 0.8, 2.5

Estimate $\int_0^2 f(x)\,dx$ using the trapezium rule.

h = 0.5

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2}[1.8 + 2.5 + 2(2.2 + 1.9 + 0.8)]$

= 3.525
Answer: 3.525

3 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 3?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(0, −1) becomes (0, 1)
(b) The maximum of the middle part is 1. Compare k = 3 with 1:
k > 1 gives 2 intersections with y = k.
Answer: (a) (0, 1) (b) 2

4 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is v = $t^2 + 8t + 6$ m s^{-1}.
Find the distance moved between t = 0 and t = 3. (The particle does not change direction.)

$s = \int_0^3 (t^2 + 8t + 6)\,dt$

$= \left[\frac{1}{3}t^3 + 4t^2 + 6t\right]$ from t = 0 to t = 3

$= 63 - 0$
Answer: 63 m

5 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the test scores for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Boys: 40 − 30 = 10. Girls: 50 − 35 = 15
(b) Medians: 35 (Boys) and 45 (Girls), so Girls has the higher median
IQRs: Girls has the larger IQR, so its scores are more spread out
Answer: (a) 10 and 15 (b) Girls has the higher median; Girls is more spread out (larger IQR), Boys more consistent

6 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = x^2e^{3-3x} - 3$.
Find the equation of the tangent to C at the point where x = 1, in the form y = mx + c.

$\frac{dy}{dx} = -x(3x - 2)e^{3-3x}$

At x = 1: y = −2 and $\frac{dy}{dx} = -1$

y + 2 = −1(x − 1), so y = −x − 1

Answer: y = −x − 1

7 Binomial critical region (All boards)

X ~ B(30, p). H_0: p = 0.25 is to be tested against H_1: p > 0.25 at the 10% significance level.
Find the critical region and the actual significance level.

Under H_0, X ~ B(30, 0.25)
P(X ≥ 12) = 0.0507 ≤ 0.1
P(X ≥ 11) = 0.1057 > 0.1
The critical region is X ≥ 12
Answer: X ≥ 12; actual significance level 5.07%

8 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = t^2 - t$, $y = 2t^3 - 3t^2$.
Find the gradient at t = −1 and the equation of the normal there.

$\frac{dx}{dt} = 2t - 1$, $\frac{dy}{dt} = 6t^2 - 6t$

Gradient = $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -4$ at t = −1

Point: (2, −5); normal gradient = $\frac{1}{4}$

$y + 5 = \frac{1}{4}(x - 2)$, so $y = \frac{1}{4}x - \frac{11}{2}$

Answer: gradient −4; normal $y = \frac{1}{4}x - \frac{11}{2}$

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 18$.

Assume $x^2 - y^2 = 18$, so (x − y)(x + y) = 18.
x − y and x + y have the same parity (their sum 2x is even).
If both are odd, the product is odd; if both are even, the product is a multiple of 4.
But 18 is even and is not a multiple of 4 (18 = 4 × 4 + 2), a contradiction.
So no such integers exist.
Answer: no integer solutions (product is odd or a multiple of 4)

10 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = −7x + 37 meets the circle $(x - 2)^2 + (y + 2)^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute y = −7x + 37: $(x - 2)^2 + (-7x + 39)^2 = 25$
$x^2 - 11x + 30 = 0$
(x − 6)(x − 5) = 0, so x = 5 or x = 6
y = −7x + 37: A = (5, 2), B = (6, −5)
$AB^2 = 1^2 + (-7)^2 = 50$
Answer: A(5, 2), B(6, −5), AB = $5\sqrt{2}$