Day 329 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 80; systematic sample of size 8.
(a) Find the sampling interval.
(b) The first item is number 8. Find the 6-th item.

(a) Interval = 80 ÷ 8 = 10
(b) 6-th item = 8 + 5 × 10 = 58
Answer: interval 10; item number 58

2 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\cos\theta = -\frac{3}{5}$ and $90° < \theta < 180°$,

find the exact value of $\tan 2\theta$.

$\sin\theta = \sqrt{1 - (-\frac{3}{5})^2} = \frac{4}{5}$

$\sin 2\theta = -\frac{24}{25}$ and $\cos 2\theta = -\frac{7}{25}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{24}{7}$

Answer: $\frac{24}{7}$

3 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train travelling at 15 m s^{-1} brakes and comes to rest with constant deceleration 5 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 15^2 - 2 \times 5 \times s$
$s = 225 \div 10 = 22.5$
(b) $0 = 15 - 5t$, so $t = 15 \div 5$
Answer: (a) 22.5 m (b) 3 s

4 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, texts sent in a day by 8 students:
12, 14, 23, 24, 32, 33, 37, 37.
Find Q_1, Q_3, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = \frac{37}{2}$, $Q_3 = 35$, IQR $= \frac{33}{2}$

Lower limit $= \frac{37}{2} - 1.5 \times \frac{33}{2} = -\frac{25}{4}$

Upper limit $= 35 + 1.5 \times \frac{33}{2} = \frac{239}{4}$

No value lies outside these limits.

Answer: $Q_1 = \frac{37}{2}$, $Q_3 = 35$, IQR $= \frac{33}{2}$; **no outliers**

5 Simplify using double angle formulae (All boards)

Simplify $\frac{1 + \cos 2x}{\sin 2x}$,

giving your answer as a single trigonometric term.

$1 + \cos 2x = 2\cos^2 x$ and $\sin 2x = 2 \sin x \cos x$

so the expression is $\frac{2\cos^2 x}{2 \sin x \cos x} = \cot x$

Answer: cot x

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6 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 200 and common ratio 0.8.
Find the smallest n for which the nth term is less than 1.

$200 \times 0.8^{n-1} < 1$
$(n - 1) \lg 0.8 < \lg(1 \div 200)$
lg 0.8 is negative, so dividing reverses the inequality:
$n - 1 > \lg(1 \div 200) \div \lg 0.8 = 23.744$, so $n > 24.744$
The smallest integer n is 25
Answer: n = 25

7 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 6 m and mass 3 kg is hinged at A and held horizontal by a light string attached at B.
The string makes an angle θ with the rod, where $\sin\theta = \frac{3}{5}$. Take g = 9.8 m s^{-2}. Find the tension in the string.

Moments about A: $T \sin\theta \times 6 = mg \times 3$
$T \times \frac{3}{5} \times 6 = 29.4 \times 3$
$T = 88.2 \div 3.6$
Answer: 24.5 N

8 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1 - u_n}$, $u_1 = \frac{1}{2}$.
(a) Show that the sequence is periodic, stating the period.
(b) Find u_{100} and $\sum_{r=1}^{100} u_r$.

(a) $u_2 = 2$, $u_3 = -1$, $u_4 = \frac{1}{2}$
$u_4 = u_1$ so the sequence repeats with period 3
(b) $100 = 3 \times 33 + 1$, so $u_{100} = u_1 = \frac{1}{2}$
Sum of one cycle = $\frac{3}{2}$, so the sum = $33 \times \frac{3}{2} + \frac{1}{2} = 50$
Answer: period 3; $u_{100} = \frac{1}{2}$; sum = 50

9 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 11 \\ 5 \\ 2 \end{pmatrix}$.
Find AB and the distance AB.

$\mathrm{AB} = \mathrm{OB} - \mathrm{OA} = \begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}$
$|\mathrm{AB}| = \sqrt{8^2 + 1^2 + 4^2} = \sqrt{81} = 9$
Answer: AB = $\begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}$, distance 9

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10 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a square number cannot be written in the form 4k + 3, where k is an integer.

Write the integer as n = 4q + j, where j = 0, 1, ..., 3.

n^2 leaves the same remainder as j^2 when divided by 4.

j = 0, 1, 2, 3 give remainders 0, 1, 0, 1.

The only possible remainders are 0, 1, never 3.

Answer: possible remainders 0, 1, never 3