Day 328 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Basic trig equations (degrees) (All boards)

Solve $\tan 2x = 0.4$ for $0 \leq x \leq 360°$, giving answers to 1 d.p.

Let $\theta = 2x$, so $\tan \theta = 0.4$ with θ in range 0° to 720°
$\theta = 21.80°, 201.80°, 381.80°, 561.80°$
$x = 10.9°, 100.9°, 190.9°, 280.9°$
Answer: $x = 10.9°, 100.9°, 190.9°, 280.9°$

2 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: mean temperature (°C) at Heathrow has mean 12, sd 3.
Outliers are more than 3 sd from the mean.
Are the values 22 and 5 outliers?

Limits: $12 \pm 3 \times 3$, so from 3 to 21
22 is outside the limits, so it is an outlier
5 is inside the limits, so it is not an outlier
Answer: 22: outlier; 5: not an outlier

3 Smooth inclined plane (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A toy car slides from rest down a line of greatest slope of a smooth plane inclined at α, where $\sin \alpha = \frac{3}{5}$ to the horizontal.

Take $g = 9.8$ m s^{-2}.
Find its acceleration, and the distance it travels in 4 s.

Along the plane: $mg \sin \alpha = ma$, so $a = 9.8 \times 0.6 = 5.88$
$s = \frac{1}{2}at^2 = \frac{1}{2} \times 5.88 \times 4^2$
Answer: 5.88 m s^{-2}; 47.04 m

4 Laws of logarithms: solving equations (All boards)

Solve $2 \ln x = \ln(x + 2)$.

$\ln x^2 = \ln(x + 2)$, so $x^2 = x + 2$
$x^2 - x - 2 = 0$, so $(x - 2)(x + 1) = 0$
$x > 0$ for $\ln x$ to exist, so $x = 2$
Answer: $x = 2$

5 Overtaking problem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sports car starts from rest and accelerates at 4 m s^{-2}. At that moment it is passed by a coach travelling at constant speed 20 m s^{-1}.
Find how long it takes to catch up, and the distance travelled.

Same distance: $\frac{1}{2} \times 4 \times t^2 = 20\ t$
$t(2\ t - 20) = 0$, $t \neq 0$ so $t = 20 \div 2$
Distance $= 20 \times 10$
Answer: 10 s, 200 m

6 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 11\ e^x + 24 = 0$,
giving your answers as exact values.

Let $y = e^x$: $y^2 - 11\ y + 24 = 0$
$(y - 3)(y - 8) = 0$, so $e^x = 3$ or $e^x = 8$
$x = \ln 3$ or $x = \ln 8$
Answer: $x = \ln 3$, $x = \ln 8$

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7 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{3x + 2}{x - 2}$ for $x \in \mathbb{R}$, $x \neq 2$.

(a) Find $f^{-1}(x)$.

(b) State the domain of f^{-1}.

Let $y = \frac{3x + 2}{x - 2}$. Then $y(x - 2) = 3x + 2$

$xy - 2y = 3x + 2$, so $x(y - 3) = 2y + 2$

$f^{-1}(x) = \frac{2x + 2}{x - 3}$

f never takes the value 3, so the domain of f^{-1} is $x \neq 3$.

Answer: (a) $f^{-1}(x) = \frac{2x + 2}{x - 3}$ (b) $x \neq 3$

8 Binomial critical region (All boards)

$X \sim B(25, p)$. H_0: $p = 0.4$ is to be tested against H_1: $p < 0.4$ at the 1% significance level.

Find the critical region and the actual significance level.

Under H_0, $X \sim B(25, 0.4)$

$P(X \leq 4) = 0.0095 \leq 0.01$

$P(X \leq 5) = 0.0294 > 0.01$

The critical region is $X \leq 4$

Answer: $X \leq 4$; actual significance level 0.95%

9 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{12}{5 + 3\sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are integers.

Multiply top and bottom by the conjugate $(5 - 3\sqrt{3})$:

Denominator: $(5 + 3\sqrt{3})(5 - 3\sqrt{3}) = 5^2 - 3^2 \times 3 = -2$

Numerator: $(12)(5 - 3\sqrt{3}) = 60 - 36\sqrt{3}$

So the fraction $= \frac{60 - 36\sqrt{3}}{-2} = -30 + 18\sqrt{3}$

Answer: $-30 + 18\sqrt{3}$

10 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of $(2 - 2x)^9$.

Term in $x^3 = 84 \times 2^6 \times (-2x)^3$

$= 84 \times 64 \times -8\,x^3$

Answer: -43008