Day 327 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $x^2 - 10x + 22$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the minimum point of $y = x^2 - 10x + 22$.

$x^2 - 10x = (x - 5)^2 - 25$
So $x^2 - 10x + 22 = (x - 5)^2 - 3$
$(x - 5)^2 \geq 0$ with equality at $x = 5$, so the minimum is -3 at $x = 5$.
Answer: (a) $(x - 5)^2 - 3$ (b) $(5, -3)$

2 Chain rule (All boards)

Given that $y = \sin^2 4x$,
find $\frac{dy}{dx}$, and the gradient of the curve when $x = \frac{\pi}{16}$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function
$\frac{dy}{dx} = 8 \sin 4x \cos 4x$
When $x = \frac{\pi}{16}$: gradient = 4
Answer: $\frac{dy}{dx} = 8 \sin 4x \cos 4x$; gradient 4

3 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the masses, m grams, of 9 eggs gives $\Sigma m = 292$ and $\Sigma m^2 = 10480$.
Find the mean and the standard deviation (3 s.f.).

Mean = $292 \div 9 = 32.4$
Variance = $\frac{10480}{9} - (32.444)^2 = 111.8$
Standard deviation = $\sqrt{111.8} = 10.6$
Answer: mean 32.4, standard deviation 10.6

4 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + 5n + 15$ is odd for all integers n.

$n^2 + 5n = n(n + 5)$
As 5 is odd, n and n + 5 have opposite parity,
so one of them is even and the product $n(n + 5)$ is even.
Even + 15 = odd, so $n^2 + 5n + 15$ is odd.
Answer: n(n + 5) is even, so the expression is odd

5 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trolley of mass 3 kg is moving on a smooth horizontal surface at 5 m s^{-1} when a constant horizontal force of 12 N, in the direction of motion, acts for 8 s.
Find the speed of the trolley at the end of this time and the distance it moves.

$F = ma$: $a = 12 \div 3 = 4$
$v = u + at = 5 + 4 \times 8$
$s = ut + \frac{1}{2}at^2$
Answer: 37 m s^{-1}; 168 m

6 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

The function f is defined by $f(x) = \arccos x$.
State (a) the domain, (b) the range of f, in radians.

arccos x is the inverse of cos x restricted to $[0, \pi]$
The domain and range swap: (a) $-1 \leq x \leq 1$, (b) $0 \leq f(x) \leq \pi$
Answer: (a) $-1 \leq x \leq 1$ (b) $0 \leq f(x) \leq \pi$

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7 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 4 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

The coefficient of friction is 0.25. Take $g = 9.8$ m s^{-2}. Find the acceleration down the plane, and the distance moved in 4 s.

$\cos\alpha = 4/5$, $\sin\alpha = 3/5$

$R = mg\cos\alpha$, $F = \mu R$

$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(3/5 - 0.25 \times 4/5)$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 3.92 \times 4^2$

Answer: 3.92 m s^{-2}; 31.36 m

8 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $5^{2x} - 30\times5^x + 125 = 0$.

Let $y = 5^x$. Then $5^{2x} = y^2$, so $y^2 - 30y + 125 = 0$

$(y - 25)(y - 5) = 0$

$y = 25$ or $y = 5$

$5^x = 25$ gives $x = 2$; $5^x = 5$ gives $x = 1$.

Answer: x = 1 or x = 2

9 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix}2\\-3\\-1\end{pmatrix}$, $\begin{pmatrix}-1\\-5\\-4\end{pmatrix}$ and $\begin{pmatrix}5\\-1\\r\end{pmatrix}$ respectively.

Given that A, B, C are collinear, find r.

$\text{AB} = \begin{pmatrix}-3\\-2\\-3\end{pmatrix}$, $\text{AC} = \begin{pmatrix}3\\2\\r - -1\end{pmatrix}$

$\text{AC} = \lambda\text{AB}$: component 1 gives $3 = \lambda \times -3$, so $\lambda = -1$

$r - -1 = -1 \times -3$, so $r = 2$

Answer: r = 2

10 Binomial mean and variance (All boards)

X has a binomial distribution with mean 16.8 and variance 5.04.

(a) Find n and p.

(b) Find P(X = 16) to 3 significant figures.

(a) $np = 16.8$ and $np(1 - p) = 5.04$

$1 - p = 5.04 \div 16.8 = 0.3$, so $p = 0.7$ and $n = 16.8 \div 0.7 = 24$

(b) $P(X = 16) = {}^{24}C_{16} \times 0.7^{16} \times 0.3^8 = 0.160$

Answer: (a) n = 24, p = 0.7 (b) 0.160