Day 326 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A company employs 32 managers, 56 technicians and 60 operators.
A stratified sample of size 37 is taken. Find the number selected from each group.

Total = 32 + 56 + 60 = 148

Sampling fraction = 37/148 = $\frac{1}{4}$

managers: 32 ÷ 4 = 8
technicians: 56 ÷ 4 = 14
operators: 60 ÷ 4 = 15
Answer: managers 8, technicians 14, operators 15

2 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (3e^{2x} + \frac{1}{x})\, dx$.

$\int (3e^{2x} + \frac{1}{x})\, dx = \frac{3e^{2x}}{2} + \ln x$

$= [\frac{3e^{2x}}{2} + \ln x]_1^{\,2}$

$= \ln 2 + \frac{3e^4}{2} - \frac{3e^2}{2}$

Answer: $\ln 2 + \frac{3e^4}{2} - \frac{3e^2}{2}$

3 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when
$3x^3 - 7x^2 - 3x - 4$ is divided by $(x - 3)$.

$3x^2 \times (x - 3) = 3x^3 - 9x^2$, leaving $2x^2 - 3x - 4$
$2x \times (x - 3) = 2x^2 - 6x$, leaving $3x - 4$
$3 \times (x - 3) = 3x - 9$, leaving 5
Quotient $3x^2 + 2x + 3$, remainder 5
Answer: quotient $3x^2 + 2x + 3$, remainder 5

4 Power, driving force and resistance (CIE)

A car of mass 800 kg moves along a horizontal road. The engine works at a constant rate of 60 kW. The resistance to motion is constant at 400 N.
Find the acceleration when the speed is 20 m s^{-1}.

P = Fv: F = 60000 ÷ 20 = 3000 N
F − R = ma: 3000 − 400 = 800 a
Answer: 3.25 m s^{-2}

5 Laws of logarithms: solving equations (All boards)

Solve $\log_3(x + 15) - \log_3(x - 1) = 1$.

$\log_3(\frac{x + 15}{x - 1}) = 1$, so $\frac{x + 15}{x - 1} = 3^1 = 3$
$x + 15 = 3(x - 1) = 3x - 3$
$2x = 18$, so $x = 9$
Answer: $x = 9$

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6 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 3)^2 + (y - 3)^2 = 169$.
The point P(−2, 15) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (3, 3). Gradient of radius CP = $\frac{12}{-5} = -\frac{12}{5}$

The tangent is perpendicular to the radius: gradient = $\frac{5}{12}$

$y - 15 = \frac{5}{12}(x + 2)$

Multiply out and collect terms: $5x - 12y + 190 = 0$

Answer: $5x - 12y + 190 = 0$

7 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.25. In a sample of 50 students, 7 pass first time.
Test, at the 5% significance level, whether the probability is different from 0.25.

H_0: p = 0.25, H_1: $p \neq 0.25$. Under H_0, X ~ B(50, 0.25)

$P(X \leq 7) = 0.0453$

Two-tailed: compare with 0.025. 0.0453 > 0.025, so do not reject H_0

There is insufficient evidence that the probability is different from 0.25

Answer: do not reject H_0: insufficient evidence that the probability is different from 0.25

8 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (4 kg) and B (6 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 8 m above the ground, then released.
Take g = 9.8 m s^{-2}.
Find the speed with which B hits the ground.

$a = (6 - 4)g \div (6 + 4) = 1.96$

$v^2 = 0 + 2 \times 1.96 \times 8 = 31.36$

$v = 5.6$

Answer: 5.6 m s^{-1}

9 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 - 3x + 1$ and the line $y = -2x + 7$ meet at the points A and B.
Find the coordinates of A and B, and the exact length AB.

$x^2 - 3x + 1 = -2x + 7$

$x^2 - x - 6 = 0$, so $(x + 2)(x - 3) = 0$

$x = -2$ or $x = 3$

$y = -2x + 7$: $y = 11$ and $y = 1$

$AB^2 = 5^2 + -10^2 = 125$

Answer: A(−2, 11), B(3, 1), AB = $5\sqrt{5}$

10 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{2x + 9}{(x + 4)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{2x + 9}{(x + 4)(x + 5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{2x + 9}{(x + 4)(x + 5)} \equiv \frac{A}{x + 4} + \frac{B}{x + 5}$, so $2x + 9 \equiv A(x + 5) + B(x + 4)$
$x = -4$: $A = 1$; $x = -5$: $B = 1$

(b) $\int_0^1 \left(\frac{1}{x + 4} + \frac{1}{x + 5}\right) dx = [\ln(x + 4) + \ln(x + 5)]_0^1$
$= \ln \frac{5}{4} + \ln \frac{6}{5} = \ln\left(\frac{3}{2}\right)$

Answer: (a) A = 1, B = 1 (b) $\ln\left(\frac{3}{2}\right)$