Day 325 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$.

Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4}$

$= [\frac{x}{2} - \frac{\sin 2x}{4}]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{4}$

Answer: $\frac{\pi}{4}$

2 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $5^{2x} - 6\times5^x + 5 = 0$.

Let $y = 5^x$. Then $5^{2x} = y^2$, so $y^2 - 6y + 5 = 0$
$(y - 5)(y - 1) = 0$
$y = 5$ or $y = 1$
$5^x = 5$ gives $x = 1$; $5^x = 1$ gives $x = 0$.
Answer: $x = 0$ or $x = 1$

3 Union, conditional probability and independence (All boards)

P(A) = 0.25, P(B) = 0.4, P(A ∩ B) = 0.05.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) P(A ∪ B) = 0.25 + 0.4 − 0.05 = 0.6

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.05}{0.4} = 0.125$

(c) P(A)P(B) = 0.25 × 0.4 = 0.1, which does not equal P(A ∩ B)
Answer: (a) 0.6 (b) 0.125 (c) not independent

4 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 60 kg stands on weighing scales in a lift. The lift decelerates at 1 m s^{-2} while moving upwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is downwards, magnitude 1
Taking up as positive: $R - 60 \times 9.8 = -60 \times 1$
$R = 60(9.8 - 1)$
Answer: 528 N

5 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find y in terms of x, given that $\frac{dy}{dx} = 3y^2e^x$

and y = 0.25 when x = 0.

$\frac{1}{y^2} dy = 3e^x dx$, so $-\frac{1}{y} = 3e^x + c$

$x = 0$, $y = \frac{1}{4}$: $-4 = 3 + c$, so $c = -7$

$\frac{1}{y} = 7 - 3e^x$, so $y = \frac{1}{7 - 3e^x}$

Answer: $y = \frac{1}{(7 - 3e^x)}$

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6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $x + y = -3$ and line l_2 has equation $5x - 2y = 20$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (2, −5)
l_1 meets y = 0 at x = −3, and l_2 meets y = 0 at x = 4
Base = 7, height = 5
Area = $\frac{1}{2}$ × base × height = $\frac{35}{2}$

Answer: $\frac{35}{2}$

7 Simplify using double angle formulae (All boards)

Simplify $\frac{\sin 2x}{1 + \cos 2x}$,
giving your answer as a single trigonometric term.

$\sin 2x = 2 \sin x \cos x$ and $1 + \cos 2x = 2 \cos^2 x$
so the expression is $\frac{2 \sin x \cos x}{2 \cos^2 x} = \tan x$

Answer: tan x

8 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 28) = 0.1587 and P(X > 64) = 0.0228.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.999$
$28 = \mu + (-0.9998)\sigma$ and $64 = \mu + 1.999\sigma$
Subtract: $36 = 2.999\,\sigma$, so $\sigma = 12.0$
$\mu = 28 - (-0.9998)(12.00) = 40.0$
Answer: μ = 40.0, σ = 12.0

9 Overtaking problem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sports car starts from rest and accelerates at 1 m s^{-2}. At that moment it is passed by a coach travelling at constant speed 12 m s^{-1}.
Find how long it takes to catch up, and the distance travelled.

Same distance: ½ × 1 × t^2 = 12 t
t(0.5 t − 12) = 0, t ≠ 0 so t = 12 ÷ 0.5
Distance = 12 × 24
Answer: 24 s, 288 m

10 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^2 + 7x \geq -3$.

Rearrange: $2x^2 + 7x + 3 \geq 0$
Factorise: $(x + 3)(2x + 1) \geq 0$
Critical values: $x = -3$ and $x = -\frac{1}{2}$
Sketch the ∪-shaped parabola: $x \leq -3$ or $x \geq -\frac{1}{2}$

Answer: $x \leq -3$ or $x \geq -\frac{1}{2}$