Day 324 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should y = 5 + x not be used for x = 15?

(a) The points lie loosely around a straight line rising: weak positive correlation
(b) The data cover x = 1 to 10; x = 15 is outside this range
so using the line would be extrapolation and may be unreliable
Answer: (a) weak positive correlation (b) extrapolation outside the range of the data

2 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(-4\mathbf{i} + \mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(4\mathbf{i} + 2\mathbf{j})$ m s^{-2}.
Find its velocity and speed when t = 4.

$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (-4\mathbf{i} + \mathbf{j}) + 4(4\mathbf{i} + 2\mathbf{j})$
$= 12\mathbf{i} + 9\mathbf{j}$
Speed $= \sqrt{(12^2 + 9^2)}$
Answer: $\mathbf{v} = (12\mathbf{i} + 9\mathbf{j})$ m s^{-1}; speed 15 m s^{-1}

3 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^2 - 2x - 4 \geq 0$.

Factorise: $2(x - 2)(x + 1) \geq 0$
Critical values: $x = -1$ and $x = 2$
Sketch the ∪-shaped parabola: $x \leq -1$ or $x \geq 2$
Answer: $x \leq -1$ or $x \geq 2$

4 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 9 pairs of values, $\bar{x} = 10$, $\bar{y} = 18$, $S_{xx} = 20$ and $S_{xy} = 50$.
(a) Find the regression line of y on x in the form y = a + bx.
(b) Estimate y when x = 14.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{50}{20} = 2.5$
$a = \bar{y} - b\bar{x} = 18 - 2.5 \times 10 = -7$
$y = -7 + 2.5x$
(b) $y = -7 + 2.5 \times 14 = 28$
Answer: $y = -7 + 2.5x$; $y = 28$

5 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = -\frac{3}{4}$ and $90° < \theta < 180°$,
find the exact value of $\tan 2\theta$.

Right-angled triangle 3, 4, 5: $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$ (θ obtuse)
$\sin 2\theta = -\frac{24}{25}$ and $\cos 2\theta = \frac{7}{25}$
$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{24}{7}$
Answer: $-\frac{24}{7}$

6 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x - 1| = |-4x - 3|$.

Either $2x - 1 = -4x - 3$ or $2x - 1 = -(-4x - 3)$
$2x - 1 = -4x - 3$ gives $x = -\frac{1}{3}$
$2x - 1 = -(-4x - 3)$ gives $x = -2$
Answer: $x = -2$ and $x = -\frac{1}{3}$

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7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-x-1}{(x+3)(x+5)}$ in partial fractions.

(b) Hence evaluate $\int_0^2 \frac{-x-1}{(x+3)(x+5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-x-1}{(x+3)(x+5)} \equiv \frac{A}{x+3} + \frac{B}{x+5}$, so $-x-1 \equiv A(x+5) + B(x+3)$
$x = -3$: $A = 1$; $x = -5$: $B = -2$

(b) $\int_0^2 \left(\frac{1}{x+3} + \frac{-2}{x+5}\right) dx = [\ln(x+3) - 2\ln(x+5)]_0^2$
$= \ln\frac{5}{3} - 2\ln\frac{7}{5} = \ln\left(\frac{125}{147}\right)$

Answer: (a) A = 1, B = −2 (b) $\ln\left(\frac{125}{147}\right)$

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers
m and n such that $21m + 35n = 142$.

Assume there are integers m and n with $21m + 35n = 142$.
$21m + 35n = 7(3m + 5n)$.
So the left-hand side is a multiple of 7.
But $142 = 7 \times 20 + 2$ is not a multiple of 7.
This is a contradiction, so no such integers exist.

Answer: no integer solutions (LHS is a multiple of 7, 142 is not)

9 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (3 kg) and B (9 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 1.25 m above the ground, then released.

Take $g = 9.8$ m s^{-2}.

Find the speed with which B hits the ground.

$a = (9 - 3)g \div (9 + 3) = 4.9$
$v^2 = 0 + 2 \times 4.9 \times 1.25 = 12.25$
$v = 3.5$

Answer: 3.5 m s^{-1}

10 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{8}{17}$ and $\sin B = \frac{5}{13}$, where A and B are acute,

find the exact value of $\sin(A + B)$.

$\cos A = \frac{15}{17}$ and $\cos B = \frac{12}{13}$
$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$= \frac{8}{17} \times \frac{12}{13} + \frac{15}{17} \times \frac{5}{13} = \frac{171}{221}$

Answer: $\frac{171}{221}$