Day 323 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix}3\\-3\\-3\end{pmatrix}$, $\begin{pmatrix}3\\0\\0\end{pmatrix}$ and $\begin{pmatrix}3\\q\\6\end{pmatrix}$ respectively.

Given that A, B, C are collinear, find q.

$\text{AB} = \begin{pmatrix}0\\3\\3\end{pmatrix}$, $\text{AC} = \begin{pmatrix}0\\q - -3\\9\end{pmatrix}$

AC = λAB: component 3 gives $9 = \lambda \times 3$, so $\lambda = 3$

$q - -3 = 3 \times 3$, so $q = 6$

Answer: q = 6

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 180; systematic sample of size 20.

(a) Find the sampling interval.

(b) The first item is number 5. Find the 4-th item.

(a) Interval = 180 ÷ 20 = 9

(b) 4-th item = 5 + 3 × 9 = 32

Answer: interval 9; item number 32

3 Product rule (All boards)

Given that $y = x^4 \ln x$,

find $\frac{dy}{dx}$, and its value when x = 1.

Product rule with $u = x^4$, $v = \ln x$

$u' = 4x^3$, $v' = \frac{1}{x}$

$\frac{dy}{dx} = u'v + uv' = x^3(4 \ln x + 1)$

When x = 1: $\frac{dy}{dx} = 1$

Answer: $\frac{dy}{dx} = x^3(4 \ln x + 1)$; value 1

4 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A remote-control car moves with constant velocity $(10\mathbf{i} - 24\mathbf{j})$ m s^{-1}, where **i** and **j** are unit vectors east and north. Find its speed and the acute angle its direction makes with **i**, to 1 d.p.

Speed = $\sqrt{(10^2 + -24^2)} = 26$

$\tan\theta = 24 \div 10$

Answer: 26 m s^{-1}; 67.4°

5 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{4}{x - 2} + 1$.

(a) Write down the asymptotes.

(b) Find where the curve crosses the axes.

(a) The denominator is zero when x = 2, and $y \to 1$ as $x \to \pm\infty$

Asymptotes: x = 2 and y = 1

(b) x = 0: $y = -2 + 1 = -1$

y = 0: $\frac{4}{x - 2} = -1$, so $x - 2 = -4$, $x = -2$

Answer: (a) x = 2, y = 1 (b) (0, −1) and (−2, 0)

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6 Normal curve and shaded area (All boards)

$X \sim N(40, 8^2)$. The shaded area on the diagram represents the probability that X lies between two values.
Find this probability to 3 significant figures.

Shaded area = P(32.00 < X < 40.00)
= P(X < 40.00) − P(X < 32.00)
= 0.5000 − 0.1587 = 0.341
Answer: 0.341

7 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 5 \\ -1 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a}\cdot\mathbf{b} = (0)(-1) + (2)(5) + (1)(-1) = 9$
$|\mathbf{a}| = \sqrt{5}$, $|\mathbf{b}| = \sqrt{27}$
$\cos\theta = \frac{9}{\sqrt{5}\sqrt{27}} = 0.7746$
$\theta = 39.2°$
Answer: 39.2°

8 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 4 kg is pulled across smooth horizontal ice by a rope with tension 5 N, inclined at α above the horizontal, where $\tan\alpha = \frac{4}{3}$.

Take g = 9.8 m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

cos α = 3/5, sin α = 4/5
(a) Horizontal: 5 × 3/5 = 4 a
(b) Vertical: R + 5 × 4/5 = 4 × 9.8
Answer: (a) 0.75 m s^{-2} (b) 35.2 N

9 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1-u_n}$, $u_1 = 4$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{40} and $\sum_{r=1}^{40} u_r$.

(a) $u_2 = -\frac{1}{3}$, $u_3 = \frac{3}{4}$, $u_4 = 4$
$u_4 = u_1$ so the sequence repeats with period 3
(b) 40 = 3 × 13 + 1, so $u_{40} = u_1 = 4$
Sum of one cycle = $\frac{53}{12}$, so the sum = $13 \times \frac{53}{12} + 4 = \frac{737}{12}$
Answer: period 3; $u_{40} = 4$; sum = $\frac{737}{12}$

10 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $2^{2x} - 12\times2^x + 32 = 0$.

Let $y = 2^x$. Then $2^{2x} = y^2$, so $y^2 - 12y + 32 = 0$
$(y - 8)(y - 4) = 0$
y = 8 or y = 4
$2^x = 8$ gives x = 3; $2^x = 4$ gives x = 2.
Answer: x = 2 or x = 3