Day 322 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 32 and the coefficient of x^2 is 448.

Find the values of n and k.

$nk = 32$, so $k = \frac{32}{n}$

$\frac{n(n-1)}{2}k^2 = 448$, so $\frac{n(n-1)}{2} \times \frac{1024}{n^2} = 448$

$\frac{(n-1) \times 1024}{2n} = 448$, so $1024(n-1) = 896n$

$n = 8$, and $k = 4$

Answer: n = 8, k = 4

2 Poisson distribution (CIE)

X ~ Po(1.5), modelling the number of flaws per metre of cloth.

Find, to 3 significant figures,

(a) P(X = 1)

(b) $P(X \leq 1)$

(a) $P(X = 1) = e^{-1.5} \times \frac{1.5^1}{1!} = 0.335$

(b) $P(X \leq 1) = P(X = 0) + ... + P(X = 1) = 0.558$

Answer: (a) 0.335 (b) 0.558

3 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x - 2| = |2x + 6|$.

Either $x - 2 = 2x + 6$ or $x - 2 = -(2x + 6)$

$x - 2 = 2x + 6$ gives $x = -8$

$x - 2 = -(2x + 6)$ gives $x = -\frac{4}{3}$

Answer: $x = -8$ and $x = -\frac{4}{3}$

4 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{40}{41}$ and $\sin B = \frac{3}{5}$, where A is obtuse and B is acute,

find the exact value of $\cos(A - B)$.

$\cos A = -\frac{9}{41}$ and $\cos B = \frac{4}{5}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= (-\frac{9}{41}) \times \frac{4}{5} + \frac{40}{41} \times \frac{3}{5} = \frac{84}{205}$

Answer: $\frac{84}{205}$

5 Work-energy: speed after falling or sliding (CIE)

A particle of mass 4 kg slides from rest down a smooth curved slope, starting 2.5 m above the bottom.

Take $g = 9.8$ m s^{-2}.

Use energy to find its speed at the bottom of the slope.

Loss of PE = gain of KE: $m \times 9.8 \times 2.5 = \frac{1}{2} m v^2$

$v^2 = 19.6 \times 2.5 = 49$

$v = 7$

Answer: 7 m s^{-1}

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6 Finding the mean or standard deviation (All boards)

The length X cm of a metal rod is normal with mean 75.
Given P(X < 86.8) = 0.975, find the standard deviation (3 s.f.).

$z = 1.960$, so $\frac{86.8 - 75}{\sigma} = 1.960$

$\sigma = 11.8 \div 1.960 = 6.02$

Answer: $\sigma = 6.02$

7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $20\cos\theta + 21\sin\theta \equiv 29\cos(\theta - 46.4°)$,
solve $20\cos\theta + 21\sin\theta = -6$ for $0 \le \theta \le 360°$. (1 d.p.)

$29\cos(\theta - 46.4°) = -6$, so $\cos(\theta - 46.4°) = \frac{-6}{29}$

$\theta - 46.4° = \pm 101.9°$

$\theta = 46.4° + 101.9° = 148.3°$

$\theta = 46.4° - 101.9° = -55.5°$, so $\theta = 304.5°$

Answer: $\theta = 148.3°, 304.5°$

8 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 3x)^{-2}$ up to the x^2 term.

(b) Use $x = 0.04$ to estimate $\frac{1}{1.12^2}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = -2$, $a = 3$:

$= 1 - 6x + 27x^2$

(b) $1 + 3 \times 0.04 = 1.12$, so substitute $x = 0.04$:

$1 - 6 \times 0.04 + 27 \times 0.04^2 = 0.80320$

Answer: (a) $1 - 6x + 27x^2 + \ldots$ (b) 0.80320

9 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 9 kg lies on a rough horizontal table (coefficient of friction 0.3). It is joined by a light string over a smooth pulley to a particle of mass 5 kg hanging freely.
Take $g = 9.8$ m s^{-2}.
The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.3 \times 9g = 9a$

Hanging: $5g - T = 5a$

Add: $5g - 2.7g = 14a$

$a = 1.61$

$T = 5(9.8 - 1.61)$

Answer: $a = 1.61$ m s^{-2}; $T = 40.95$ N

10 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = x^2 + 2$ to find the exact value of

$\int_0^3 \frac{2x}{x^2 + 2}\,dx$.

$\frac{du}{dx} = 2x$, so $2x\,dx = du$

Limits: $x = 0$ gives $u = 2$, $x = 3$ gives $u = 11$

$\int_2^{11} \frac{1}{u}\,du = [\ln u]_2^{11}$

$= \ln(\frac{11}{2})$

Answer: $\ln(\frac{11}{2})$