Day 321 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-2, 8)$ and $(8, -2)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{-2+8}{2}, \frac{8+(-2)}{2}\right) = (3, 3)$

(b) Gradient $= \frac{-10}{10} = -1$

$y - 8 = -(x + 2)$
Multiply out and collect terms: $x + y - 6 = 0$
Answer: (a) (3, 3) (b) $x + y - 6 = 0$

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 10 cm and arc length 18 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

$s = r\theta$, so $\theta = \frac{18}{10} = 1.8$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 10^2 \times 1.8 = 90$

Answer: (a) 1.8 rad (b) 90 cm^2

3 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(2\mathbf{i} - 5\mathbf{j})$ N and $(6\mathbf{i} - \mathbf{j})$ N act on a particle of mass 5 kg.
Find the magnitude of the acceleration of the particle.

Resultant $= (2\mathbf{i} - 5\mathbf{j}) + (6\mathbf{i} - \mathbf{j}) = 8\mathbf{i} - 6\mathbf{j}$
$|R| = \sqrt{(8^2 + -6^2)} = 10$ N
$a = F/m = 10 \div 5$
Answer: 2 m s^{-2}

4 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^3 + 5n + 7$ is odd for all integers n.

$n^3 + 5n = n(n^2 + 5)$
If n is even, $n(n^2 + 5)$ is even.
If n is odd, n^2 is odd and $n^2 + 5$ is even, so $n(n^2 + 5)$ is even.
So $n^3 + 5n$ is always even, and adding 7 gives an odd number.
Answer: expression is odd

5 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trolley of mass 5 kg is moving on a smooth horizontal surface at 4 m s^{-1} when a constant horizontal force of 6 N, in the direction of motion, acts for 2 s.
Find the speed of the trolley at the end of this time and the distance it moves.

F = ma: $a = 6 \div 5 = 1.2$
$v = u + at = 4 + 1.2 \times 2$
$s = ut + \frac{1}{2}at^2$
Answer: 6.4 m s^{-1}; 10.4 m

6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, a quadratic.
(a) Find the coordinates of the local maximum point of $y = |f(x)|$.
(b) How many solutions has $|f(x)| = 2$?

(a) $y = |f(x)|$ reflects the part of the curve below the x-axis in the x-axis:
$(2, -1)$ becomes $(2, 1)$
(b) The maximum of the middle part is 1. Compare $k = 2$ with 1:
$k > 1$ gives 2 intersections with $y = k$.
Answer: (a) (2, 1) (b) 2

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7 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\cot^2 x - \operatorname{cosec} x - 8 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $2\operatorname{cosec}^2 x - \operatorname{cosec} x - 10 = 0$

$(2\operatorname{cosec} x - 5)(\operatorname{cosec} x + 2) = 0$, so $\operatorname{cosec} x = \frac{5}{2}$ or -2

$\operatorname{cosec} x = \frac{5}{2}$, so $\sin x = \frac{2}{5}$: $x = 23.6°, 156.4°$

$\operatorname{cosec} x = -2$, so $\sin x = -\frac{1}{2}$: $x = 210.0°, 330.0°$

Answer: x = 23.6°, 156.4°, 210.0°, 330.0°

8 Binomial: smallest n (All boards)

Given that each lottery ticket wins a prize independently with probability 0.1, find the least number of tickets needed for the probability of at least one success to exceed 0.9.

P(at least one) $= 1 - 0.9^n > 0.9$

$0.9^n < 0.1$

$n > \frac{\log 0.1}{\log 0.9} = 21.85$

Answer: n = 22

9 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The height of the tide in a harbour is modelled by $H = 12 + 6\sin(\frac{\pi t}{12})$, where t is in hours after midnight.

Find (a) the greatest and least values, (b) the values of t with $0 \le t \le 24$ when it equals 9 m.

(a) sin ranges from -1 to 1: greatest $12 + 6 = 18$, least $12 - 6 = 6$

(b) $12 + 6\sin(\frac{\pi t}{12}) = 9$, so $\sin(\frac{\pi t}{12}) = -\frac{1}{2}$

$\frac{\pi t}{12} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, so $t = 14$ or 22

Answer: (a) 18 m, 6 m (b) t = 14 and 22

10 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass, X g, of a loaf is normally distributed with mean 20 and s.d. 6.
A random sample of 4 is taken. Find the probability that the mean mass is less than 17 (3 s.f.).

$\overline{X} \sim N(20, \frac{6^2}{4})$, standard deviation 3.000

$z = \frac{17 - 20}{3.000} = -1.000$

P = 0.159

Answer: 0.159