Day 320 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

6 values of x are coded using $y = \frac{(x - 20)}{2}$.

$\Sigma y = 10$ and $\Sigma y^2 = 94$.

Find the mean and standard deviation of x (3 s.f.).

Mean of y = 10 ÷ 6 = 1.667

Mean of x = 20 + 2 × 1.667 = 23.3

Variance of y = $\frac{94}{6} - (1.667)^2 = 12.89$

SD of y = 3.590, so SD of x = 2 × 3.590 = 7.18

Answer: mean 23.3, standard deviation 7.18

2 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = -4x - 1$ for $-3 < x < 3$.

Find the range of f.

f is decreasing, so the extreme values occur at the ends of the domain.

f(−3) = 11 and f(3) = −13

x = −3 is excluded and x = 3 is excluded, so f = 11 is excluded and f = −13 is excluded.

Answer: $-13 < f(x) < 11$

3 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int e^{4x}\,dx$,

(b) the exact value of $\int_0^1 e^{4x}\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{e^{4x}}{4} + c$

(b) $\left[\frac{e^{4x}}{4}\right]_0^1$

$= \frac{e^4}{4} - \frac{1}{4}$

Answer: (a) $\frac{e^{4x}}{4} + c$ (b) $\frac{e^4}{4} - \frac{1}{4}$

4 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.

(a) Find the coordinates of the local maximum point of y = |f(x)|.

(b) How many solutions has |f(x)| = 4?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:

(1, −4) becomes (1, 4)

(b) The maximum of the middle part is 4. Compare k = 4 with 4:

k = 4 gives 3 intersections with y = k.

Answer: (a) (1, 4) (b) 3

5 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The time, X minutes, to complete a task is normally distributed with mean 50 and s.d. 4.

A random sample of 16 is taken. Find the probability that the mean time is more than 51 (3 s.f.).

$\overline{X} \sim N(50, \frac{4^2}{16})$, standard deviation 1.000

$z = \frac{51 - 50}{1.000} = 1.000$

P = 0.159

Answer: 0.159

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6 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{4}{5}$ and $\sin B = \frac{15}{17}$, where A and B are acute,
find the exact value of cos(A + B).

$\cos A = \frac{3}{5}$ and $\cos B = \frac{8}{17}$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$= \frac{3}{5} \times \frac{8}{17} - \frac{4}{5} \times \frac{15}{17} = -\frac{36}{85}$

Answer: $\mathbf{-\frac{36}{85}}$

7 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $2x - y = 4$ and line l_2 has equation $4x - 5y = -4$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (4, 4)

l_1 meets y = 0 at x = 2, and l_2 meets y = 0 at x = −1

Base = 3, height = 4

Area = $\frac{1}{2}$ × base × height = 6

Answer: 6

8 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\cot^2 x - \operatorname{cosec} x - 4 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $2\operatorname{cosec}^2 x - \operatorname{cosec} x - 6 = 0$

(cosec x − 2)(2 cosec x + 3) = 0, so cosec x = 2 or $-\frac{3}{2}$

cosec x = 2, so sin x = $\frac{1}{2}$: x = 30.0°, 150.0°

cosec x = $-\frac{3}{2}$, so sin x = $-\frac{2}{3}$: x = 221.8°, 318.2°

Answer: x = 30.0°, 150.0°, 221.8°, 318.2°

9 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $(-2\mathbf{i} - 3\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(4\mathbf{i} + 4\mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(-2\mathbf{i} - 3\mathbf{j}) + \mathbf{P} = 3(4\mathbf{i} + 4\mathbf{j}) = (12\mathbf{i} + 12\mathbf{j})$

$\mathbf{P} = 14\mathbf{i} + 15\mathbf{j}$

$|\mathbf{P}| = \sqrt{(14^2 + 15^2)}$

Answer: $\mathbf{P} = (14\mathbf{i} + 15\mathbf{j})$ N; $|\mathbf{P}| = 20.5$ N

10 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A golf ball is projected from ground level with initial velocity components 20 m s^{-1} horizontally and 29.4 m s^{-1} vertically upwards.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 29.4^2 - 2 \times 9.8 \times H$, H = 864.36 ÷ 19.6

(b) At the top the vertical velocity is zero, so the speed is the horizontal component.

Answer: (a) 44.1 m (b) 20 m s^{-1}